

Estimation of Strength of Different Population Groups Existing In Andhra Pradesh – A deductive approach

D V Ramana Murty*, G Arti and M.VivekanandaMurty*****

**Department of Statistics, VT College, Rajahmundry, East Godavari District, Andhra Pradesh, India.*

E-mail: drdvrmurthy@gmail.com

*** Department of Management, GITAM (Deemed to be University), Visakhapatnam, India.*

E-mail: artiglen2012@gmail.com

*and ***Former Professor of Statistics, Andhra University, Visakhapatnam, India.*

E-mail: vmushini@gmail.com

Abstract

This paper is on the deductive way of estimating the population percentage of different groups Existing in the population. This was developed and presented as it will be easy and give more relevant estimation and need of the government of Andhra Pradesh, India for ready reference. It is based on census reports and earlier reports of different commissions appointed by the State and Central Governments. In this paper estimating percentage of one particular important population group in Andhra Pradesh was estimated by the deductive approach mentioned.

Keywords: Population percentage, Census, SC, ST, BC and OC population, Characteristics of Backwardness.

1. INTRODUCTION:

Government of Andhra Pradesh want to know whether a particular group/caste existing in the newly formed state of Andhra Pradesh is possessing backward class characteristics or not. This information will help the administration to recommend that caste to be included in the list of Backward Classes or not. Since majority of these characteristics are directly or indirectly dependent on the percentage of population existing in the State of Andhra Pradesh, this method was suggested as there is an urgent need of knowing the percentage of different population groups become necessary, which was not available earlier. Hence this paper was developed for finding the estimated population Percentages of the group/caste that was in need.

After bifurcation of the combined Andhra Pradesh, newly formed Andhra Pradesh was formed in 2014 consists of 13 districts namely Srikakulam, Vizianagaram, Visakhapatnam, East Godavari, West Godavari, Krishna, Guntur, Prakasam, Nellore, Chittoor, Kadapa, Kurnool and Anantapur. As per the 2011 census total population was identified as 49386799 out of which 8445398 were SCs and 2631145 were STs. That is 17.10 percent of the total population is SCs and 5.33 percent of the total population is STs. But for different reasons Government needs to know further classification of population based on caste and social status. But due to unknown reasons that information was not available even to the government agencies as well as common public.

But in recent past Andhra Pradesh Government has appointed one Commission namely Manjunath Commission for the purpose of identifying some caste to be included in the list of Backward Classes especially one particular Caste namely Kapu (Kapu, Telaga, Balija and Ontari in Andhra Pradesh), who are presently not in the listed Backward Classes. Here the actual problem is that how to establish backwardness of a community.

2. CHARACTERISTICS IDENTIFIED FOR BACKWARD CLASSES:

The Mandal Commission (1980-1990) adopted various methods and techniques to collect the necessary data and evidence. To identify who qualified as an "other backward class (OBC)," the commission identified different criteria which could be grouped under three major headings namely social, educational and economic.

2.1 Social: Castes/classes considered as socially backward by others if they possess the following characteristics.

- 1) Castes/classes which mainly depend on manual labor for their livelihood,
- 2) Castes/classes where at least 25 per cent females and 10 per cent males above the state average get married at an age below 17 years in rural areas and at least 10 per cent females and 5 per cent males do so in urban areas.
- 3) Castes/classes where participation of females in work is at least 2 per cent and above the state average.

2.2 Educational: Castes/classes considered as Educationally backward by others if they possess the following characteristics.

- 1) Castes/classes where the number of children in the age group of 5-15 years who never attended school is at least 25 per cent above the state average.
- 2) Castes/classes when the rate of student drop-out in the age group of 5-15 years is at least 25 per cent above the state average,
- 3) Castes/classes amongst whom the proportion of matriculates is at least 25 per cent below the state average,

2.3 Economic: Castes/classes where the average value of family assets is at least 25 per cent below the state average,

- 1) Castes/classes where the number of families living in kuccha houses is at least 25 per cent above the state average,
- 2) Castes/classes where the source of drinking water is beyond half a kilometer for more than 50 per cent of the households,
- 3) Castes/classes where the number of households having taken consumption loans is at least 25 per cent above the state average.

2.4 Weighting indicators: As the above three groups are not of equal importance for the purpose, separate weightage was given to indicators in each group. All the Social indicators were given a weightage of 3 points each, educational indicators were given a weightage of 2 points each and economic indicators were given a weightage of 1 point each. Economic, in addition to Social and Educational Indicators, were considered important as they directly flowed from social and educational backwardness. This also helped to highlight the fact that socially and educationally backward classes are economically backward also.

2.5 Other Commissions reports: In addition to the above other sociologists and other Backward Class Commissions appointed by the Government from time to time like K N Anantharaman Commission, Muralidhar Rao Commission, Puttuswamy commission, National BC Commission identified some more criteria for inclusion in the Backward Classes list. Following are some of the important criteria that communities should possess for getting the Backwardness status.

- 1) Communities are not allowed to enter into the houses of the affluent people. That is one-way untouchables and nomadic, seminomadic and vimukthijatis.
- 2) Communities moving from one place to another place for the sake of earning food
- 3) Communities depending on profession like weaving, goldsmith, today tapping etc.
- 4) SCs converted to Christianity
- 5) Social customs of divorce and remarriage among their women.
- 6) Women in their community should go to agricultural and other labor works and helping the family in their agricultural works.
- 7) The less education levels among them.
- 8) Less representation of their community in positions like people's representatives (MLAs, MPs, ZP Chairmen, Municipal chairmen, Councilors, ZPTC members etc.), IAS, IPS, Group I, II officers, Vice Chancellors, Executive members of the universities, Professors, Associate Professors,

Assistant Professors in the Universities (both private and government), Scientists. Less in the sense that less than their population share in the total population etc.

- 9) Castes which are Culturally backward.
- 10) Communities in which Child labor exists.

For testing whether the characteristics that are mentioned above, in majority of the cases proportional to population of different compositions of the population (Groups/Castes) is required, which are not generally available.

This paper is to concentrate on estimation of percent population belongs to different groups based on different casts and social status.

3. ESTIMATE OF POPULATION PERCENTAGE OF DIFFERENT COMPONENTS OF POPULATION IN AP:

After bifurcation of the States the newly formed residual Andhra Pradesh remains with only 13 districts. The population figure as per Census 2011 are presented in table 1.

Table 1. Distribution of District Population of AP and SC and ST population according to 2011 census

District	Population 2011 census	Population 2011 census		Percentage of population to Total	
		SC	ST	SC	ST
Srikakulam	2703114	255664	166118	9.4581	6.1454
Vizagapatnam	2344474	247728	235556	10.5664	10.0472
Visakhapatnam	4290589	329486	618500	7.6792	14.4152
East Godavari	5154296	945269	213195	18.3394	4.1362
West Godavari	3936966	811698	109072	20.6173	2.7704
Krishna	4517398	871063	132464	19.2824	2.9323
Guntur	4887813	957407	247089	19.5876	5.0552
Prakasam	3397448	787861	151145	23.1897	4.4487
Nellore	2963557	666588	285997	22.4928	9.6504
Chittur	4174064	785760	159165	18.8248	3.8131
Kadapa	2882469	465794	75886	16.1595	2.6326
Karnool	4053463	737945	82831	18.2052	2.0434
Ananthapur	4081148	583135	154127	14.2885	3.7765
Total AP	49386799	8445398	2631145	17.1005	5.3276

As per Census 2011, Hindus consisting of 88.5%, Muslims 9.6% and others 1.9%. Out of 88.5% Hindus SC Population of Andhra Pradesh is 17.1% and ST population of Andhra Pradesh is 5.3%. (From Table 1)

Therefore, AP Hindu Population percentage in total population other than SC and ST are $88.5 - 17.1 - 5.3 = 66.1\%$

A small overlapping between Muslims and Muslims in BCs, but can be ignored as it is very small. An SC candidate whose religion was changed to Christian Religion will come under B.C- 'C'. Religious converts from other castes in Hindus is also very negligible and they will be treated under the changed religion only.

In the absence of a credible survey on Back ward classes, Andhra Pradesh has been relying on data called from four sources namely (a) the caste-wise census of 1931; (b) the Mandal Commission report (c) the Socio-Educational Economic Survey of Castes & Communities of Andhra Pradesh (SEESCCAP); and (d) a multi-purpose household survey to enumerate persons eligible for ration cards. The SEESCCAP, though inconclusive in the end, was commissioned in 1994 to survey the entire population and compare the relative progress of various communities and determine the actual percentage of the BC population in Andhra Pradesh, assumed to be 52 per cent. Some organization claim that it is more than and some claim that it is only 48%. On the average, it is estimated that BC population is 50%.

The Total population other than SC, ST and BC = $66.1 - 50 = 16.1\%$

That is 16.1 % are castes consists of Kapus, Kshatriyas, Brahmins, Vysyas, Reddys, Kammas, Velamas, etc. This clearly show that the Kapus (including Kapus, Balijas, Ontari, Telaga in OC category) who are aspiring for social backwardness cannot exceed 8% (Fifty percent of the population other than SC, ST and BCs of the AP) as it covers in all most all districts and some district like east and west Godavari and Krishna districts were dominated by Kapus. May be in some districts they are represented in good number, but state as whole cannot be more than 8%. (Excluding toorpu Kapus and Gajula Kapus who are already included in the list of BCs). Similarly, one can get a rough estimate, which is sufficient for administrative purpose for all the remaining castes and total of all of them cannot be more than 8 percent.

4. UTILITY OF PERCENTAGE OF POPULATION:

The type of estimation given in section 3, will help in getting an approximate percentage of different groups based on castes in Andhra Pradesh and in many situations this rough estimation will suffice to serve the purpose. In this contest, once the percentage of population was estimated it is obvious to see whether a particular group / caste is socially, politically and educationally backward or not, by comparing this percent with other percentages of representation of that group/caste (easily available) in education, occupation of different administrative position and possessing honors like IAS, IPS etc., representation in people's representatives like MPs, MLAs, MLCs etc. Percentage of representation in the services like IAS, IPS officers, High Court Judges and other judiciary officers, Central Service Officers, Professors and

other Teaching and Research personals in the universities, colleges and other research institutes, Group I, Group II and Group III officers in the government, both Central and State Government contractors, Owners of Educational institutes, Real estate businessmen, Executive Council members to various Universities nominated by the Government of Andhra Pradesh are also to be compared with the population percentage of that group/ caste in deciding the backward class nature of that group/ caste.

CONCLUSION:

Hence the above deductive method was suggested by taking one community as example for estimating their percentage in total population. This was presented with a special reference to the present-day problem facing by the Government/ related organizations. The estimated proportion will suffice the need of the administrator.

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Half Logistic - Pareto Distribution in Acceptance Sampling Plans Based on Truncated Life Tests

S. Bhanu Prakash, Sd. Jilani, D. Madhu Latha, D. Naga Purnima, V. Lakshmi Sailaja

Abstract— This paper proposes an innovative acceptance sampling plan under a truncated life test framework, assuming that the lifetime of a product follows a Half Logistic-Pareto Distribution (HLPD). For different acceptance numbers, confidence levels, and ratios of test duration to the true mean lifetime, the minimum required sample sizes are determined to validate the specified mean life. The corresponding operating characteristic (OC) values and producer's risk associated with the proposed plan are also evaluated. Comprehensive tables are provided, and the plan's performance is illustrated and interpreted through detailed numerical examples.

Index Terms—Acceptance sampling, Operating characteristic function, Half Logistic - Pareto distribution, Producer's risk, Consumer's risk, truncated life test

I. INTRODUCTION

The acceptance sampling plans are employed to ensure the acceptability of a product lot, allowing the consumer to decide whether to accept or reject it based on a randomly selected sample. The procedure begins with identifying the minimum sample size required to ensure a specified average life when the life test concludes at a predetermined time. These types of tests are known as truncated life tests. In modern quality control, acceptance sampling plans play a vital role, as they assist manufacturers in reducing variability and enhancing the overall quality of their products.

Several researchers have explored acceptance sampling plans based on truncated life tests for different lifetime distributions. For instance, studies such as [9] and [23] examined the generalized exponential distribution, while [25] investigated the Rayleigh distribution. A three-parameter kappa distribution was analyzed in [6], and [5] developed a sampling plan for the exponentiated Fréchet distribution under truncated life tests. Additionally, [14] focused on the log-logistic distribution, Sriramachandran and Palanivel explored the exponentiated inverse Rayleigh

distribution, and [7] proposed time-truncated acceptance sampling plans using the generalized inverted exponential distribution.

Traditional lifetime distributions such as the Weibull, Logistic, and Lomax models exhibit limited flexibility in representing diverse real-world phenomena. These distributions often fail to provide an adequate fit across various practical scenarios. To overcome such limitations, numerous researchers have recently proposed new families of statistical models using innovative approaches. Several techniques for developing flexible lifetime models have been discussed in the literature [15]. For instance, [2] introduced the Exponentiated Lomax distribution, while [4] developed the Discrete Poisson-Lomax distribution by employing the Lomax model as a mixing distribution for the Poisson parameter. The Beta-Pareto model was proposed by [3], and [1] presented both the Uniform Exponential Distribution (UED) and the Half Logistic-Pareto Distribution (HLPD). The term *Generalized Pareto Distribution (GPD)* was first used by Sharma et al. [20] in the context of statistical inference for the upper tail of a distribution. Further, several compound lifetime models have been proposed—such as the Half Logistic Generalized Weibull Distribution by [8] and the Half Logistic Lomax Distribution for lifetime modeling by [16]. The Poisson-Lomax model was formulated by [11] through compounding Poisson and Lomax distributions. A two-parameter Lindley distribution was proposed by [22] using a mixture of exponential (q) and gamma ($2, q$) components. Moreover, [18] developed acceptance sampling plans based on the Topp-Leone Gompertz distribution, whereas [19] designed plans for the three-parameter inverted Topp-Leone model. [10] introduced the Zeghdoudi distribution for acceptance sampling under truncated life tests using real data applications. Recent works include research on the mechanism of technological innovation investment and performance [26], the T-distribution random nearest neighbor embedding combined with entropy weight [27], and an optimization model for trail billet cutting under dual subjective-objective constraints [28].

II. THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS OF THE HLPD

In this section, a new two-parameter distribution named the Half Logistic Pareto Distribution (HLPD) is introduced, characterized by the new Scale (β) and Shape (δ) parameters. The corresponding probability density function (pdf), cumulative distribution function (cdf), and survival function of the proposed HLPD are defined as follows.

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S. Bhanu Prakash is an Assistant Professor of Freshmen Engineering Department, Aditya University, Surampalem-533437, A.P., India (corresponding author e-mail: prakash9293165890@gmail.com).

Sd. Jilani is an Assistant Professor of Department of Statistics, Acharya Nagarjuna University, Guntur-522510, A.P., India (e-mail: jilanisyed1992@gmail.com).

D. Madhu Latha is a Lecturer in Statistics of SKVT Government Degree College, Rajahmundry-533103, A.P., India (e-mail: koppisitemadhu1atha@gmail.com).

D. Naga Purnima is an Assistant Professor of Freshmen Engineering Department, Aditya University, Surampalem-533437, A.P., India (e-mail: munukutla1977@gmail.com).

V. Lakshmi Sailaja is an Assistant Professor of Freshmen Engineering Department, Aditya University, Surampalem-533437, A.P., India (e-mail: sailajavaradhi@gmail.com).

A random variable $X \sim \text{HLPD}(\beta, \delta)$ has probability density function in the form

$$f(x; \beta, \delta) = \frac{2\delta\beta^\delta x^{\delta-1}}{[x^\delta + \beta^\delta]^2}; \quad x > 0, (\beta, \delta) > 0 \quad (1)$$

A random variable $X \sim \text{HLPD}(\beta, \delta)$ correspond Cumulative density function is in the form

$$F(x; \beta, \delta) = \frac{x^\delta - \beta^\delta}{x^\delta + \beta^\delta} \quad (2)$$

A random variable $X \sim \text{HLPD}(\beta, \delta)$ then Survival function is given by

$$S(x; \beta, \delta) = \frac{2\beta^\delta}{x^\delta + \beta^\delta} \quad (3)$$

A random variable $X \sim \text{HLPD}(\beta, \delta)$ then Hazard function is given by

$$H(x; \beta, \delta) = \frac{\delta x^{\delta-1}}{x^\delta + \beta^\delta} \quad (4)$$

Here (β, δ) are Scale and shape parameters.

III. ACCEPTANCE SAMPLING PLAN

This section presents the development of acceptance sampling plans based on the **Half Logistic-Pareto Distribution (HLPD)**. To date, no prior research has focused on constructing acceptance sampling plans using this distribution.

A truncated life test-based acceptance sampling plan is characterized by the following components:

- (1) **Sample size (n)**: The number of units placed on test.
- (2) **Acceptance number (c)**: the lot is accepted if c or fewer failures occurred within the test duration t .
- (3) **Maximum test duration (t)**: the predetermined time at which the test concludes.
- (4) **Ratio (t/σ_0)**: the proportion between the test time and the specified average life σ_0 .

A. Minimum Sample Size

Assuming the lot size is sufficiently large to be treated as infinite, the probability of lot acceptance is derived using the binomial distribution. Here, the problem is to determine the minimum sample size n essential to satisfy the inequality

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - \alpha^* \quad (5)$$

up to an acceptance number c for given values of $\alpha^* \in (0, 1)$

where $p = F(t; \sigma_0)$ is the probability of a failure observed within the time t which depends only on the ratio t/σ_0 .

If the number of observed failures within the time t is at most c , then from Inequality (5) we can confirm with probability P that $F(t; \sigma) \leq F(t; \sigma_0)$, which implies $\sigma \leq \sigma_0$.

The smallest sample sizes satisfying the Inequality (5) for $t/\sigma_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$, $\alpha^* = 0.75, 0.9, 0.95, 0.99$ and $c = 0, 1, 2, \dots, 10$ are presented in TABLE I and TABLE II. The values of t/σ_0 and α^* considered in this study are the same values which are considered in [13], [12], [14], [5] and [21].

When the value of $p = F(t; \sigma_0)$ is small and the sample size n is large, the binomial distribution can be approximated by Poisson distribution with parameter $\nu = np$. Accordingly, the left side of (5) can be

$$\sum_{i=0}^c \frac{e^{-\nu} \nu^i}{i!} \leq 1 - \alpha^* \quad (6)$$

Where $\nu = nF(t; \sigma_0)$. The minimum values of n satisfying equation (6) are obtained for the same combination of $\beta, \delta, \alpha^*, t/\sigma_0$ and c values as those used for equation (5). The results are given in TABLE III and TABLE IV.

B. Operating characteristic of the sampling plan

The operating characteristic (OC) function of the sampling plan $(n, c, t/\sigma_0)$ is the probability of accepting a lot. It is given as

$$L(p) = \sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \quad (7)$$

where $p = F(t; \sigma_0)$. It can be seen that operating characteristic is an increasing function of σ_0 . For a given $\alpha^*, t/\sigma_0$, the selection of c and n is determined by the operating characteristics. TABLE V provides values of the operating characteristics as a function of σ/σ_0 for selected sampling plans based on binomial probabilities, specified for $\beta = 1.5$ and $\delta = 2.0$. Meanwhile, TABLE III offers similar values based on Poisson probabilities. However, since these values do not differ significantly from those in TABLE I, the operating characteristic values for Poisson-based plans are not presented separately.

C. Producer's risk

In life testing data used for quality control sampling plans, lots with an average life exceeding the specified threshold $\sigma > \sigma_0$ are considered good lots. Here, the quality parameter is the population average life of the product—an example of a “larger-the-better” quality characteristic. That is, any lot whose product average life surpasses the specified value is treated as acceptable. However, applying a sampling plan can sometimes lead to incorrect decisions, similar to errors encountered in classical hypothesis testing. The probability of mistakenly rejecting a good lot is known as the producer's risk. For example, if the producer's risk is set at 0.05, we aim to identify the range of values σ/σ_0 (based on the ratio of the specified average

life to the unknown true average life) that will ensure the risk of rejecting a good lot does not exceed 5%, assuming a particular sampling plan is used. We know that the distribution function of the population distribution can be written as a function of σ/σ_0 .

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \geq 0.95 \quad (8)$$

For a given sampling plan $(n, c, t/\sigma_0)$ on the basis of binomial probabilities and specified confidence level α^* , the minimum values of σ/σ_0 satisfying the inequality equation (8) are given in TABLE VI. Poisson probabilities are not much different from binomial probabilities and hence, those are not presented.

IV. ILLUSTRATION OF THE TABLES AND EXAMPLE FOR SAMPLING PLAN

Suppose the lifetime of a product follows a Half Logistic - Pareto distribution and the goal is to demonstrate that the true mean life is at least 1000 hours. Setting the consumer's confidence level at $1-\alpha^*=0.95$, the test is designed to end at $t=1000$ hours. For an acceptance number $c=2$, the required sample size n can be obtained from TABLE II, corresponding to the values of $1-\alpha^*=0.95$, $t/\sigma_0 = 1.257$ and $c=2$, which gives $n=26$. Therefore, 26 units must be tested. If within 1000 hours, no more than two failures occur among the tested units, the experimenter can conclude with 95% confidence that the mean life is at least 1000 hours. Using the Poisson approximation to the binomial distribution under the same conditions, TABLE IV yields a sample size of $n=29$.

For the sampling plan $(n=12, c=2, t/\sigma_0=1.257)$ under Half Logistic - Pareto distribution, the operating characteristic values from TABLE V.

σ/σ_0	2	4	6	8	10	12
$L(p)$	0.414 4	0.811 7	0.922 3	0.961 3	0.978 0	0.986 4

This shows that, if the true mean life is twice the specified mean life ($\sigma/\sigma_0=2$) the producer's risk is approximately 0.228.

From TABLE VI, we can get the value of σ/σ_0 for various choices of c , t/σ_0 in order that the producer's risk may not exceed 0.05. Thus, in the above example we obtain the values of $\sigma/\sigma_0 = 7.22$. That is, the product should have an average life of 7.22 times the specified average life 1000 hours in order that under the above acceptance sampling plan $(n=26, c=2, t/\sigma_0=1.257)$, the product be accepted with probability of at least 0.95.

A. Real Data Example

In this section, we present the analysis of real data, partially considered in [17] for illustrative purposes. The data represent the Glass Fiber Strength data: 1.29, 1.31, 1.32, 1.33, 1.34, 1.36, 1.37, 1.39, 1.40, 1.41, 1.42, 1.43, 1.44, 1.46, 1.47, 1.48, 1.50, 1.52, 1.53, 1.54, 1.55, 1.56, 1.58, 1.59, 1.60, 1.62, 1.63, 1.64, 1.65, 1.67, 1.68, 1.70, 1.72, 1.73, 1.74, 1.76, 1.78, 1.80, 1.82, 1.85, 1.87, 1.90, 1.93, 1.95, 1.98, 2.00, 2.05, 2.10, 2.15, 2.20, 2.25, 2.30, 2.35, 2.40, 2.45, 2.50, 2.55, 2.60, 2.70, 2.80, 2.90.

The MLEs of the parameters of Type II GHLD based on the waiting times data are obtained as $\hat{\beta}=1.3$ and $\hat{\delta}=5.0$. To test the goodness of fit we apply the Kolmogorov-Smirnov test, we found that the maximum distance between the observed and fitted Glass Fiber Strength is 0.0934 with p -value 0.6276. Thus HLPD provides a reasonable fit for the Glass Fiber Strength. To emphasize the goodness of fit we also plot histogram and superimposed empirical density and theoretical density. Further, goodness of fit is Glass Fiber Strength with Q-Q plot, both plots are displayed in the Figure 1.

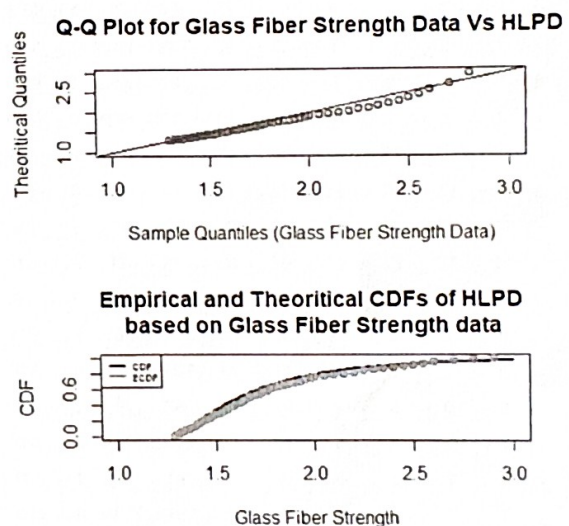


Fig. 1. Histogram with superimposed theoretical density Plot and Q-Q Plot of the fitted HLP Distribution for the Glass Fiber Strength.

V. CONCLUSIONS

In this study, acceptance sampling plans are developed for the Half Logistic-Pareto distribution, focusing on the mean life when the test is terminated at a predetermined time. A systematic procedure is outlined for constructing the proposed sampling plans based on the mean of the Half Logistic-Pareto distribution. These plans can be effectively applied to verify whether the product's average life exceeds a specified standard. Additionally, several tables are presented to assist practitioners in implementing and illustrating the construction of such acceptance sampling plans using real data.

TABLE I
UNITS FOR MAGNETIC PROPERTIES MINIMUM SAMPLE SIZES
NECESSARY TO ASSERT THE AVERAGE LIFE TO EXCEED A GIVEN
VALUE σ_0 , WITH PROBABILITY $1 - \alpha^*$, AND THE CORRESPONDING
ACCEPTANCE NUMBER C, USING BINOMIAL PROBABILITIES

$1 - \alpha^*$	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	0	0	8	4	2	2	1	1
0.75	1	4	22	15	8	4	3	3	3
0.75	2	NULL	NULL	23	11	6	5	4	4
0.75	3	7	37	29	15	8	7	6	5
0.75	4	NULL	NULL	36	19	10	8	7	7
0.75	5	10	51	43	22	12	10	9	8
0.75	6	NULL	NULL	50	25	14	11	10	9
0.75	7	13	65	56	29	16	13	11	11
0.75	8	NULL	NULL	63	32	18	14	13	12
0.75	9	17	79	69	35	20	16	14	13
0.75	10	NULL	NULL	76	39	22	17	15	14
0.9	0	NULL	NULL	13	6	3	2	2	2
0.9	1	4	23	22	11	6	4	4	3
0.9	2	NULL	NULL	30	15	8	6	5	5
0.9	3	7	38	38	19	10	8	7	6
0.9	4	NULL	NULL	45	23	12	10	8	8
0.9	5	11	52	53	27	14	11	10	9
0.9	6	NULL	NULL	60	30	16	13	11	10
0.9	7	14	66	67	34	18	15	13	12
0.9	8	NULL	NULL	74	38	21	16	14	13
0.9	9	17	81	81	41	23	18	16	14
0.9	10	NULL	NULL	88	45	25	19	17	16
0.95	0	NULL	NULL	17	8	4	3	3	2
0.95	1	4	23	26	13	7	5	4	4
0.95	2	NULL	NULL	35	17	9	7	6	5
0.95	3	8	38	44	22	11	9	8	7
0.95	4	NULL	NULL	52	26	14	11	9	8
0.95	5	11	52	60	30	16	12	11	10
0.95	6	NULL	NULL	67	34	18	14	12	11
0.95	7	14	67	75	37	20	16	14	12
0.95	8	NULL	NULL	82	41	22	17	15	14
0.95	9	17	81	89	45	24	19	17	15
0.95	10	NULL	NULL	97	49	26	21	18	17
0.99	0	NULL	NULL	25	12	6	4	4	3
0.99	1	4	24	37	18	9	7	6	5
0.99	2	NULL	NULL	47	23	12	9	7	7
0.99	3	8	38	56	27	14	11	9	8
0.99	4	NULL	NULL	65	32	17	13	11	10
0.99	5	11	53	73	36	19	14	12	11
0.99	6	NULL	NULL	82	41	21	16	14	13
0.99	7	14	67	90	45	24	18	15	14
0.99	8	NULL	NULL	98	49	26	20	17	15
0.99	9	17	81	106	53	28	21	19	17
0.99	10	NULL	NULL	114	57	30	23	20	18

TABLE II
MINIMUM SAMPLE SIZES NECESSARY TO ASSERT THE AVERAGE LIFE TO
EXCEED A GIVEN VALUE σ_0 , WITH PROBABILITY $1 - \alpha^*$, AND THE
CORRESPONDING ACCEPTANCE NUMBER C, USING BINOMIAL
PROBABILITIES

$1 - \alpha^*$	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	0	0	6	3	2	1	1	1
0.75	1	4	17	12	6	3	3	2	2
0.75	2	NULL	NULL	17	9	5	4	4	3
0.75	3	6	28	22	11	7	5	5	5
0.75	4	NULL	NULL	27	14	8	7	6	6
0.75	5	9	39	32	17	10	8	7	7
0.75	6	NULL	NULL	37	19	11	9	9	8
0.75	7	12	50	42	22	13	11	10	9
0.75	8	NULL	NULL	47	24	14	12	11	10
0.75	9	14	61	52	27	16	13	12	11
0.75	10	NULL	NULL	57	30	17	14	13	13
0.9	0	NULL	NULL	10	5	2	2	2	1
0.9	1	4	18	16	8	4	3	3	3
0.9	2	NULL	NULL	22	11	6	5	4	4
0.9	3	6	29	28	14	8	6	6	5
0.9	4	NULL	NULL	34	17	10	8	7	6
0.9	5	9	40	40	20	11	9	8	8
0.9	6	NULL	NULL	45	23	13	10	9	9
0.9	7	12	51	50	26	15	12	11	10
0.9	8	NULL	NULL	56	28	16	13	12	11
0.9	9	14	62	61	31	18	14	13	12
0.9	10	NULL	NULL	66	34	19	16	14	13
0.95	0	NULL	NULL	12	6	3	2	2	2
0.95	1	4	18	20	10	5	4	3	3
0.95	2	NULL	NULL	26	13	7	5	5	4
0.95	3	6	29	32	16	9	7	6	6
0.95	4	NULL	NULL	39	19	11	8	7	7
0.95	5	9	40	44	22	12	10	9	8
0.95	6	NULL	NULL	50	25	14	11	10	9
0.95	7	12	51	56	28	16	13	11	10
0.95	8	NULL	NULL	61	31	17	14	12	12
0.95	9	15	62	67	34	19	15	14	13
0.95	10	NULL	NULL	72	37	21	17	15	14
0.99	0	NULL	NULL	19	9	4	3	3	2
0.99	1	4	18	27	13	7	5	4	4
0.99	2	NULL	NULL	35	17	9	7	6	5
0.99	3	7	29	41	20	11	8	7	7
0.99	4	NULL	NULL	48	24	13	10	9	8
0.99	5	9	41	55	27	14	11	10	9
0.99	6	NULL	NULL	61	30	16	13	11	10
0.99	7	12	52	67	33	18	14	13	12
0.99	8	NULL	NULL	73	36	20	16	14	13
0.99	9	15	63	79	39	22	17	15	14
0.99	10	NULL	NULL	85	42	23	18	16	15

TABLE III
MINIMUM SAMPLE SIZES NECESSARY TO ASSERT THE AVERAGE LIFE TO
EXCEED A GIVEN VALUE σ_0 , WITH PROBABILITY $1 - \alpha^*$, AND THE
CORRESPONDING ACCEPTANCE NUMBER C, USING POISSON
PROBABILITIES

$1 - \alpha^*$	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	NULL	NULL	9	5	3	2	2	2
0.75	1	3	21	16	9	5	4	4	4
0.75	2	NULL	NULL	24	13	7	6	6	5
0.75	3	5	34	31	16	10	8	7	7
0.75	4	NULL	NULL	37	20	12	10	9	8
0.75	5	7	47	44	23	14	11	10	10
0.75	6	NULL	NULL	51	27	16	13	12	11
0.75	7	9	60	58	30	18	14	13	12
0.75	8	NULL	NULL	64	34	20	16	14	14
0.75	9	10	73	71	37	22	18	16	15
0.75	10	NULL	NULL	77	40	23	19	17	16
0.9	0	NULL	NULL	14	8	5	4	3	3
0.9	1	3	22	23	12	7	6	6	5
0.9	2	NULL	NULL	32	17	10	8	7	7
0.9	3	5	35	40	21	12	10	9	9
0.9	4	NULL	NULL	48	25	15	12	11	10
0.9	5	7	49	55	29	17	14	13	12
0.9	6	NULL	NULL	62	33	19	16	14	13
0.9	7	9	61	70	37	21	17	16	15
0.9	8	NULL	NULL	77	40	23	19	17	16
0.9	9	10	74	84	44	26	21	19	18
0.9	10	NULL	NULL	91	48	28	23	20	19
0.95	0	NULL	NULL	18	10	6	5	4	4
0.95	1	3	22	28	15	9	7	7	6
0.95	2	NULL	NULL	38	20	12	10	9	8
0.95	3	5	36	46	24	14	12	11	10
0.95	4	NULL	NULL	54	29	17	14	12	12
0.95	5	7	49	62	33	19	16	14	13
0.95	6	NULL	NULL	70	37	21	18	16	15
0.95	7	9	62	78	41	24	19	18	16
0.95	8	NULL	NULL	85	45	26	21	19	18
0.95	9	10	75	93	49	28	23	21	20
0.95	10	NULL	NULL	100	52	30	25	22	21
0.99	0	NULL	NULL	28	15	9	7	6	6
0.99	1	3	23	40	21	12	10	9	9
0.99	2	NULL	NULL	50	26	15	13	11	11
0.99	3	5	36	60	31	18	15	14	13
0.99	4	NULL	NULL	69	36	21	17	16	15
0.99	5	7	49	78	41	24	19	17	16
0.99	6	NULL	NULL	86	45	26	21	19	18
0.99	7	9	62	95	50	29	24	21	20
0.99	8	NULL	NULL	103	54	31	26	23	22
0.99	9	10	75	111	58	34	28	25	23
0.99	10	NULL	NULL	119	62	36	29	27	25

TABLE IV
MINIMUM SAMPLE SIZES NECESSARY TO ASSERT THE AVERAGE LIFE TO
EXCEED A GIVEN VALUE σ_0 , WITH PROBABILITY $1 - \alpha^*$, AND THE
CORRESPONDING ACCEPTANCE NUMBER C, USING POISSON
PROBABILITIES

$1 - \alpha^*$	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	NULL	NULL	11	7	4	2	2	2
0.75	1	3	16	12	7	4	4	4	3
0.75	2	NULL	NULL	18	10	6	5	5	5
0.75	3	4	26	23	13	8	7	6	6
0.75	4	NULL	NULL	28	15	10	8	8	7
0.75	5	5	36	34	18	11	10	9	9
0.75	6	NULL	NULL	39	21	13	11	10	10
0.75	7	7	45	44	23	14	12	12	11
0.75	8	NULL	NULL	49	26	16	14	13	12
0.75	9	8	55	53	29	18	15	14	14
0.75	10	NULL	NULL	58	31	19	16	15	15
0.9	0	NULL	NULL	11	6	4	3	3	3
0.9	1	3	17	18	10	6	5	5	5
0.9	2	NULL	NULL	24	13	8	7	7	6
0.9	3	4	27	30	16	10	9	8	8
0.9	4	NULL	NULL	36	19	12	10	10	9
0.9	5	5	37	42	22	14	12	11	11
0.9	6	NULL	NULL	47	25	16	13	12	12
0.9	7	7	46	53	28	17	15	14	13
0.9	8	NULL	NULL	58	31	19	16	15	15
0.9	9	8	56	64	34	21	18	17	16
0.9	10	NULL	NULL	69	37	23	19	18	17
0.95	0	NULL	NULL	14	8	5	4	4	4
0.95	1	3	17	22	12	7	6	6	6
0.95	2	NULL	NULL	29	15	10	8	8	7
0.95	3	4	27	35	19	12	10	9	9
0.95	4	NULL	NULL	41	22	14	12	11	11
0.95	5	5	37	47	25	16	13	12	12
0.95	6	NULL	NULL	53	28	18	15	14	13
0.95	7	7	47	59	32	19	17	15	15
0.95	8	NULL	NULL	65	35	21	18	17	16
0.95	9	8	56	70	38	23	20	18	18
0.95	10	NULL	NULL	76	41	25	21	20	19
0.99	0	NULL	NULL	21	11	7	6	6	6
0.99	1	3	17	30	16	10	9	8	8
0.99	2	NULL	NULL	38	20	13	11	10	10
0.99	3	4	27	45	24	15	13	12	11
0.99	4	NULL	NULL	52	28	17	15	14	13
0.99	5	6	37	59	31	19	17	15	15
0.99	6	NULL	NULL	65	35	21	18	17	16
0.99	7	7	47	72	38	24	20	19	18
0.99	8	NULL	NULL	78	42	26	22	20	20
0.99	9	8	56	84	45	28	24	22	21
0.99	10	NULL	NULL	90	48	29	25	23	23

TABLE V
OPERATING CHARACTERISTIC VALUES OF THE SAMPLING PLAN
($n, c, t/\sigma$) FOR $\lambda (1-\alpha^*)$, UNDER HALF LOGISTIC - PARETO
DISTRIBUTION $\beta = 1.5, \delta = 2.0$

$1-\alpha^*$	n	t/σ	2	4	6	8	10	12
0.75	23	1.257	0.6859	0.9264	0.9730	0.9873	0.9931	0.9958
	11	1.571	0.7232	0.9405	0.9790	0.9903	0.9948	0.9969
	6	2.356	0.7242	0.9446	0.9810	0.9914	0.9954	0.9973
	5	3.141	0.6745	0.9330	0.9771	0.9897	0.9945	0.9967
	4	3.927	0.7279	0.9487	0.9830	0.9925	0.9961	0.9977
	4	4.712	0.6513	0.9288	0.9760	0.9893	0.9943	0.9967
0.90	30	1.257	0.5204	0.8640	0.9466	0.9740	0.9855	0.9911
	15	1.571	0.5275	0.8722	0.9510	0.9765	0.9870	0.9921
	8	2.356	0.5246	0.8786	0.9549	0.9788	0.9884	0.9930
	6	3.141	0.5347	0.8867	0.9590	0.9810	0.9897	0.9938
	5	3.927	0.5519	0.8954	0.9629	0.9830	0.9909	0.9946
	5	4.712	0.4558	0.8590	0.9485	0.9761	0.9871	0.9922
0.95	35	1.257	0.4144	0.8117	0.9223	0.9613	0.9780	0.9864
	17	1.571	0.4386	0.8311	0.9326	0.9670	0.9815	0.9886
	9	2.356	0.4345	0.8387	0.9376	0.9700	0.9834	0.9899
	7	3.141	0.4098	0.8319	0.9356	0.9693	0.9831	0.9898
	6	3.927	0.3971	0.8290	0.9351	0.9692	0.9832	0.9898
	5	4.712	0.4558	0.8590	0.9485	0.9761	0.9871	0.9922
0.99	47	1.257	0.2220	0.6727	0.8493	0.9203	0.9532	0.9703
	23	1.571	0.2330	0.6929	0.8626	0.9286	0.9585	0.9739
	12	2.356	0.2279	0.7034	0.8714	0.9345	0.9625	0.9766
	9	3.141	0.2231	0.7078	0.8756	0.9374	0.9644	0.9779
	7	3.927	0.2746	0.7543	0.9004	0.9512	0.9727	0.9833
	7	4.712	0.1897	0.6859	0.8665	0.9331	0.9621	0.9766

TABLE VI
MINIMUM RATIO OF TRUE MEAN LIFE TO SPECIFIED MEAN LIFE FOR THE
ACCEPTABILITY OF A LOT WITH PRODUCER'S RISK OF 0.05
($\beta = 1.5, \delta = 2.0$)

$1-\alpha^*$	c	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	26.85	26.59	25.38	33.90	20.51	23.24
0.75	1	7.08	7.30	6.57	6.31	7.54	8.54
0.75	2	4.70	4.29	4.17	4.49	4.05	4.58
0.75	3	3.54	3.50	3.30	3.75	3.69	3.26
0.75	4	3.05	3.08	2.85	2.89	2.89	3.28
0.75	5	2.74	2.68	2.57	2.74	2.86	2.75
0.75	6	2.54	2.42	2.39	2.36	2.47	2.40
0.75	7	2.35	2.32	2.25	2.32	2.20	2.50
0.75	8	2.24	2.71	2.15	2.09	2.25	2.27
0.75	9	2.12	2.05	2.07	2.08	2.07	2.09
0.75	10	2.06	2.02	2.00	1.92	1.92	1.95
0.9	0	43.57	39.80	37.91	33.90	40.51	45.91
0.9	1	10.47	10.17	10.22	8.77	10.48	8.54
0.9	2	6.17	5.96	5.77	5.57	5.36	6.07
0.9	3	4.67	4.50	4.26	4.40	4.48	4.18
0.9	4	3.83	3.77	3.52	3.80	3.45	3.91
0.9	5	3.40	3.33	3.08	3.09	3.28	3.24

TABLE VI
MINIMUM RATIO OF TRUE MEAN LIFE TO SPECIFIED MEAN LIFE FOR
THE ACCEPTABILITY OF A LOT WITH PRODUCER'S RISK OF 0.05
($\beta = 1.5, \delta = 2.0$)

$1-\alpha^*$	c	1.257	1.571	2.356	3.141	3.927	4.712
0.9	6	3.06	2.94	2.79	2.91	2.81	2.80
0.9	7	2.82	2.75	2.59	2.78	2.77	2.82
0.9	8	2.64	2.61	2.58	2.48	2.49	2.55
0.9	10	2.39	2.35	2.33	2.23	2.30	2.39
0.95	0	56.95	53.01	50.45	50.64	60.52	45.91
0.95	1	12.40	12.08	12.04	11.21	10.48	11.88
0.95	2	7.22	6.79	6.56	6.63	6.65	6.07
0.95	3	5.42	5.24	4.73	5.04	5.26	5.08
0.95	4	4.44	4.29	4.18	4.25	4.00	3.91
0.95	5	3.86	3.72	3.58	3.43	3.69	3.71
0.95	6	3.43	3.35	3.19	3.18	3.15	3.19
0.95	7	3.17	3.01	2.92	3.00	3.05	2.82
0.95	8	2.94	2.82	2.72	2.68	2.73	2.82
0.95	9	2.76	2.68	2.56	2.59	2.69	2.58
0.95	10	2.64	2.57	2.44	2.52	2.48	2.60
0.99	0	83.70	79.43	75.51	67.37	80.52	68.58
0.99	1	17.71	16.85	15.66	16.07	16.31	15.18
0.99	2	9.74	9.28	8.93	8.76	7.93	8.98
0.99	3	6.93	6.49	6.16	6.32	6.03	5.96
0.99	4	5.57	5.32	5.17	5.14	5.08	5.14
0.99	5	4.72	4.50	4.33	4.11	4.10	4.18
0.99	6	4.21	4.08	3.79	3.73	3.81	3.94
0.99	7	3.81	3.69	3.58	3.45	3.32	3.45
0.99	8	3.52	3.41	3.28	3.25	3.20	3.09
0.99	9	3.30	3.19	3.05	2.92	3.10	3.05
0.99	10	3.12	3.01	2.86	2.82	2.84	2.81

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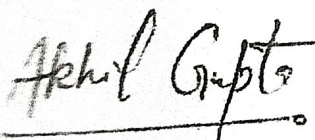
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DV Ramana Murty
Department of Statistics,
VT College, Rajahmundry,
East Godavari District,
Andhra Pradesh, India

G Arti
Department of Management,
JNTUAM (Deemed to be
University), Visakhapatnam,
Andhra Pradesh, India

M Vivekananda Murty
Professor of Statistics,
Andhra University,
Visakhapatnam,
Andhra Pradesh, India

Two component mixture of Laplace and Laplace type distributions with applications to manpower planning models

DV Ramana Murty, G Arti and M Vivekananda Murty

Abstract

This paper is on the mixtures of Laplace and Laplace type bimodal distribution with the probability p and $1-p$ respectively. In addition to the distribution properties and inferential aspects studied the same distribution was applied to the real-life data on the manpower planning of an organization under the two different promotional policies namely promotion by seniority and promotion by random.

Keywords: Laplace, two parameter Laplace, bimodal, mixtures, man power planning models, promotion by seniority, promotion by random

1. Introduction

In two parameters Laplace type distributions and three parameter Laplace type distributions. Distributions it was observed that the point at $X = \mu$ is zero, which is seemingly not useful in some practical situations. So to lift the drift at the mean point μ one way of consideration is assuming that the random variable follows Laplace distribution with probability p and two parameter Laplace type distribution with probability $1-p$ ($0 \leq p \leq 1$). This idea is imbedded in generating the mixture distributions. Hence in this paper it was developed the probability density function of two component mixture of Laplace and Laplace type distributions. The distributional properties of this distribution are also studied. This was applied to a practical manpower situation.

2. Mixtures of Laplace and Laplace Type Distributions

Let us consider a random variable X , which is such that, with probability p , it comes from Laplace distribution with probability density function

$$f(x, \mu, \beta) = \frac{1}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|}, -\infty < x < \infty, -\infty < \mu < \infty, \beta > 0$$

And with probability $1-p$ comes from a two parameter Laplace type distribution with probability density function

$$f(x, \mu, \beta) = \frac{1}{4\beta} \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|}, -\infty < x < \infty, -\infty < \mu < \infty, \beta > 0$$

The probability density function of X is

$$\begin{aligned} f(x, \mu, \beta, p) &= \frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta} \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \\ &= \left[\frac{2p+(1-p)\left(\frac{x-\mu}{\beta}\right)^2}{4\beta} \right] e^{-\left|\frac{x-\mu}{\beta}\right|}, -\infty < x < \infty, -\infty < \mu < \infty, \beta > 0 \end{aligned}$$

Correspondence
DV Ramana Murty
Department of Statistics,
VT College, Rajahmundry,
East Godavari District,
Andhra Pradesh, India

$$\mu_0 = 0, \mu_2 = 2\beta^2, \mu_3 = 0, \mu_4 = \beta^4(360 - 366p)$$

→8

Hence the variance of the distribution is $2\beta^2(6 - 5p)$. Since the distribution is symmetric, its skewness is zero and the kurtosis of the distribution is

$$\beta_2 = \frac{(360 - 366p)}{4(6 - 5p)^2}$$

→9

Hazard rate of the distribution

The hazard rate of the distribution is $h(x) = \frac{f(x)}{1-F(x)}$

→10

Where $f(x)$ and $F(x)$ are as given in equations (1) and (2) respectively.

Additive property of the distribution

For verify the additive property of the distribution, consider two independent variables X_1 and X_2 with density function

$$f(x_1, \mu_1, \beta_1, p) = \frac{p}{2\beta_1} e^{-\left|\frac{x_1 - \mu_1}{\beta_1}\right|} + \frac{1-p}{4\beta_1} \left(\frac{x_1 - \mu_1}{\beta_1}\right)^2 e^{-\left|\frac{x_1 - \mu_1}{\beta_1}\right|}$$

$$f(x_2, \mu_2, \beta_2, p) = \frac{p}{2\beta_2} e^{-\left|\frac{x_2 - \mu_2}{\beta_2}\right|} + \frac{1-p}{4\beta_2} \left(\frac{x_2 - \mu_2}{\beta_2}\right)^2 e^{-\left|\frac{x_2 - \mu_2}{\beta_2}\right|}$$

The characteristic functions are

$$\phi_{X_1}(t) = p \left[\frac{e^{it\mu_1}}{1 + \beta_1^2 t^2} \right] + (1-p) \left[\frac{e^{it\mu_1}}{2} \left\{ \frac{2 + 6\beta_1^2 t^2}{(1 - \beta_1^2 t^2)^3} \right\} \right] \text{ And}$$

$$\phi_{X_2}(t) = p \left[\frac{e^{it\mu_2}}{1 + \beta_2^2 t^2} \right] + (1-p) \left[\frac{e^{it\mu_2}}{2} \left\{ \frac{2 + 6\beta_2^2 t^2}{(1 - \beta_2^2 t^2)^3} \right\} \right]$$

Then the characteristic function of the sum of the random variables, $X_1 + X_2$ is

$$\phi_{X_1+X_2}(t) = \phi_{X_1}(t)\phi_{X_2}(t)$$

$$= \left\{ \frac{e^{it\mu_1}}{2} \left[\frac{p}{1 + \beta_1^2 t^2} + (1-p) \left\{ \frac{2 + 6\beta_1^2 t^2}{(1 - \beta_1^2 t^2)^3} \right\} \right] \right\} \left\{ \frac{e^{it\mu_2}}{2} \left[\frac{p}{1 + \beta_2^2 t^2} + (1-p) \left\{ \frac{2 + 6\beta_2^2 t^2}{(1 - \beta_2^2 t^2)^3} \right\} \right] \right\}$$

The lack of closed form of the characteristic function does not permit to have a single way of identifying the distribution of the sum of the random variables. That is additive property was not possessing by this distribution. However, from Cramer 1955 one can invent the characteristic function by using the inversion formula

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{itx} \phi_X(t) dt$$

4. Estimation of the Parameters

This section is on the estimation of the parameters arising from the mixture of Laplace and Laplace type distribution. The drift parameter p (mixing parameter) can be estimated by using the method of moments along with the location and scale parameters. Since the estimation of the location and scale parameter involves the drift parameter, estimate p can be obtained by equating the sample kurtosis with the population kurtosis.

We have $\mu_4 = \beta^4(360 - 366p)$ and $\mu_2 = 2\beta^2(6 - 5p)$

Therefore p can be estimated by solving the following quadratic equation and picking up the value which lie between 0 and 1 as an estimate of the parameter

$$p = \frac{\mu_4}{\mu_2^2} = \frac{n \sum_{i=1}^n (x_i - \bar{x})^4}{\left[\sum_{i=1}^n (x_i - \bar{x})^2 \right]^2} = \frac{(40 - 366p)}{(16 - 5p)^2}$$

→11

Where the RHS is the sample Kurtosis. The moment estimators μ and β^2 can be obtained as conditional to p as $\hat{\mu} = \bar{x}$ and

$$2\beta^2(6 - 5p) = s^2$$

contains two parameters μ and β . To maintain the parsimonious property of the modal in two components of the mixture population, the same parameters are considered. If $p = 1$, this distribution becomes Laplace distribution with parameters μ and β . And if $p = 0$, the distribution becomes two parameter Laplace type distributions.

3. Properties of the Distribution

The distribution function of the random variable X is

$$\begin{aligned}
 F_X(x) &= \int_{-\infty}^x f(x) dx = \int_{-\infty}^x \left[\frac{p}{2\beta} e^{-\frac{|x-\mu|}{\beta}} + \frac{1-p}{4\beta} \left(\frac{x-\mu}{\beta} \right)^2 e^{-\frac{|x-\mu|}{\beta}} \right] dx \\
 &= p \left[1 - \frac{e^{-\frac{|x-\mu|}{\beta}}}{2} \right] + (1-p) \left[1 - \left\{ \frac{1}{2} + \left(\frac{x-\mu}{2\beta} \right) + \left(\frac{x-\mu}{2\beta} \right)^2 \right\} e^{-\frac{|x-\mu|}{\beta}} \right], \text{ for } x \geq \mu \\
 &= p \left[\frac{e^{-\frac{|x-\mu|}{\beta}}}{2} \right] + (1-p) \left[\frac{1}{2} + \left(\frac{x-\mu}{2\beta} \right) + \left(\frac{x-\mu}{2\beta} \right)^2 \right] e^{-\frac{|x-\mu|}{\beta}}, \text{ for } x < \mu
 \end{aligned}$$

→2

For different values of μ , β and p , the frequency curves of the distribution are plotted and shown in figure: 1

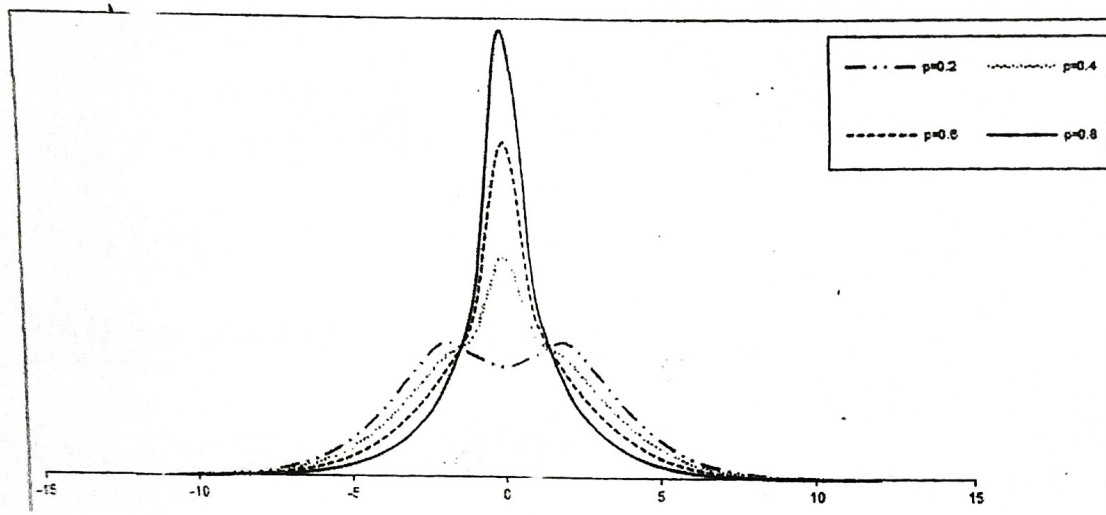


Fig 1: The frequency curves of the distribution

Mean, Mode and median of the distribution

The mean of this random variable X is

$$\begin{aligned}
 E(x) &= \int_{-\infty}^{\infty} x f(x) dx = \int_{-\infty}^{\infty} x \left[\frac{p}{2\beta} e^{-\frac{|x-\mu|}{\beta}} + \frac{1-p}{4\beta} \left(\frac{x-\mu}{\beta} \right)^2 e^{-\frac{|x-\mu|}{\beta}} \right] dx \\
 &= \frac{p}{2\beta} \int_{-\infty}^{\infty} x e^{-\frac{|x-\mu|}{\beta}} dx + \frac{1-p}{4\beta} \int_{-\infty}^{\infty} x \left(\frac{x-\mu}{\beta} \right)^2 e^{-\frac{|x-\mu|}{\beta}} dx \\
 &= p\mu + (1-p)\mu = \mu
 \end{aligned}$$

→3

This distribution is a bimodal distribution for the value of $0 \leq p \leq \frac{1}{3}$. For $p = \frac{1}{3}$, this is unimodal. For $0 < p < \frac{1}{3}$, the modes are

$$\mu - \beta \left(1 + \sqrt{1 - \frac{2p}{1-p}} \right) \text{ And } \mu - \beta \left(1 + \sqrt{1 - \frac{2p}{1-p}} \right)$$

The distance between two modes is $2 \left[\beta \left(1 + \sqrt{1 - \frac{2p}{1-p}} \right) \right]$

→4

The median of this distribution can be obtained by solving the equation

$$\int_{-\infty}^x f(x) dx + \int_x^{\infty} f(x) dx = 1$$

$$(6-5p) = s^2 \Rightarrow \hat{\beta} = \frac{s^2}{2(6-5p)}$$

→12

where, s^2 is the sample variance.

The variance of these estimators is given by $V(\bar{x}) = \frac{2\beta^2(6-5p)}{n}$

$$V(s^2) = \frac{\mu_4 - \mu_2^2}{n} - \frac{2(\mu_4 - 2\mu_2^2)}{n^2} + \frac{(\mu_4 - 3\mu_2^2)}{n^3}$$

→13

Where μ_i is the i th central moment

Therefore,

$$V(s^2) = \left[\frac{(360-366p)\beta^4 - 4(6-5p)^2}{n} \right] - 2 \left[\frac{(360-366p)\beta^4 - 8\beta^4(6-5p)^2}{n^2} \right] + \left[\frac{(360-366p)\beta^4 - 12\beta^4(6-5p)^2}{n^3} \right]$$

As these variances tends to zero as $n \rightarrow \infty$, these estimators $\bar{x}$ and s^2 are consistent for μ and σ^2 .

The maximum likelihood estimation

The maximum likelihood estimation of p and β^2 can be obtained after estimating the value of μ through median similar to the location parameter estimated in two parameter Laplace type distributions. After substituting the estimate of μ the likely hood equation of the sample is

$$L = \prod_{i=1}^n f(x_i; \mu, \beta, p) = \left(\frac{p}{2\beta}\right)^n + \left(\frac{1-p}{4\beta}\right)^n \prod_{i=1}^n \left[\left(\frac{x_i - \mu}{\beta}\right)^2 \right] e^{-\sum_{i=1}^n \left| \frac{x_i - \mu}{\beta} \right|}$$

$$\log L = -n \log p - n \log 2\beta + n \log(1-p) - n \log 4\beta + 2 \sum_{i=1}^n \left(\frac{x_i - \mu}{\beta} \right) - \sum_{i=1}^n \left| \frac{x_i - \mu}{\beta} \right|$$

Differentiating with respect to p and β^2 the likelihood equations are

$$\frac{\partial \log L}{\partial p} = \frac{n}{p} - \frac{n}{(1-p)} = 0$$

$$\frac{\partial \log L}{\partial \beta} = \frac{-n}{2\beta} - \frac{n}{4\beta} - \frac{n}{2\beta^2} \sum (x_i - \mu) + \frac{1}{\beta^2} \sum |x_i - \mu|$$

$$\text{Let } \beta^2 = \theta \Rightarrow \beta = \sqrt{\theta}$$

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta^2} - \sum (x_i - \mu) \theta^{-3/2} + \frac{1}{2} \sum |x_i - \mu| \theta^{-3/2}$$

→14

Respectively.

Solving these equations iteratively for p and β^2 using the Numerical techniques like Newton Raphson's Method, we can obtain the maximum likely hood estimators.

5. Application of mixture of Laplace and two Parameters Laplace type bimodal distribution to Manpower Planning

In this section we consider the application of the developed distribution to the manpower planning models. The complete length of service of an employee in an organization is the total duration of time that employee spends in an organization. In general, the complete length of service can be viewed as a random variable as it is determined by a number of factors in nature. This is one of the most important features characterizing the manpower situation and is important in manpower models, which describe the manpower structure of the organization and useful in obtaining optimal man power policies.

Several researchers have approximated the complete length of service distributions by different probability distributions. Silcock (1954) [10] have considered that the complete length of service of an employee fallows and exponential distribution. However exponential distribution is unimodal having long right tail with constant hazard rate. But in many private and public-sector organizations the frequency curve of the complete length of service of an employee in an organization gradually increase up to a point and decline up to some other point and again increase and decrease, that is the curve is bimodal. This phenomenon may be due to the imposition of probation period in many organizations. After an employee joining the organization, because of social and behavioral factors he is leaving and his complete length of service forms bimodal. Since the complete length of service of an employee cannot be negative, and cannot be greater than a specified value A, we can truncate the theoretical distribution given in section -3 at '0' in left and at A in right and impose the condition that the location parameter $\mu > 0$. Then we can approximate the complete length of service of an employee fallows left truncated mixture of Laplace and Laplace type distribution.

$$\frac{p}{2} \left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \right) + \frac{(1-p)}{2} \left\{ 1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \left(1 - \left(\frac{t-\mu}{\beta}\right) + \frac{1}{2} \left(\frac{t-\mu}{\beta}\right)^2 \right) \right\} \text{ for } t \geq \mu$$

The force of separation, which is known as rate of labour wastage can be obtained as

$$L(t) = \frac{g(t)}{g(t)}$$

Therefore, for this manpower model the rate labour wastage is

$$= \frac{\left[\frac{2p + (1-p) \left(\frac{x-\mu}{\beta}\right)^2}{4\beta} \right] e^{-\left|\frac{x-\mu}{\beta}\right|}}{\left[M - \left\{ \frac{p}{2} \left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \right) + \frac{(1-p)}{2} \left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \left(1 - \left(\frac{t-\mu}{\beta}\right) + \frac{1}{2} \left(\frac{t-\mu}{\beta}\right)^2 \right) \right\} \right]} \text{ for } t < \mu$$

$$= \frac{2p + (1-p) \left(\frac{x-\mu}{\beta}\right)^2}{4\beta \left[M - \left\{ \frac{p}{2} \left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \right) + \frac{(1-p)}{2} \left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \left(1 - \left(\frac{t-\mu}{\beta}\right) + \frac{1}{2} \left(\frac{t-\mu}{\beta}\right)^2 \right) \right\} \right]} \text{ for } t \geq \mu$$

→19

Where

$$M = \left\{ p \left[1 - \frac{e^{-\left(\frac{A-\mu}{\beta}\right)}}{2} \right] + (1-p) \left[\frac{1}{2} + \left(\frac{A-\mu}{2\beta}\right) + \left(\frac{A-\mu}{2\beta}\right)^2 \right] e^{-\left(\frac{A-\mu}{\beta}\right)} \right\} - \left[\frac{1}{2} + (1-p) \left(\frac{\mu^2}{4\beta^2} - \frac{\mu}{2} \right) \right] e^{-\frac{\mu}{\beta}}$$

The average time that an employee spends in the organization is

$$E(X) = \int_0^A t g(t) dt = \int_0^A t \frac{1}{M} \left[\frac{2p + (1-p) \left(\frac{x-\mu}{\beta}\right)^2}{4\beta} \right] e^{-\left|\frac{x-\mu}{\beta}\right|} dt$$

$$= \frac{1}{M} \left[\frac{p}{2\beta} \int_0^A t e^{-\left|\frac{x-\mu}{\beta}\right|} dt + \frac{(1-p)}{4\beta} \int_0^A t \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} dt \right]$$

Using the transformation and on simplification, one can get

$$\frac{1}{M} \left\{ \frac{p}{2} \left[\left[e^{\frac{\mu}{\beta}} (\beta - \mu) - e^{-\left(\frac{A-\mu}{\beta}\right)} (\beta + A - \mu) \right] + \mu \left(e^{\frac{\mu}{\beta}} - e^{-\left(\frac{A-\mu}{\beta}\right)} \right) \right] \right.$$

$$+ \frac{(1-p)}{4} \left[\left[e^{\frac{\mu}{\beta}} \left(6\beta - 6\mu + \frac{\mu^2}{\beta} - \frac{\mu^3}{\beta^2} \right) - e^{-\left(\frac{A-\mu}{\beta}\right)} \left(6\beta + 6(A-\mu) + \frac{3(A-\mu)^2}{\beta} - \frac{(A-\mu)^3}{\beta^2} \right) \right] + \mu e^{\frac{\mu}{\beta}} \left(\frac{\mu^2}{\beta^2} + \frac{2\mu}{\beta} + 1 \right) - e^{-\left(\frac{A-\mu}{\beta}\right)} \left(\left(\frac{A-\mu}{\beta}\right)^2 + \left(\frac{A-\mu}{\beta}\right) + 1 \right) \right] \right\}$$

→20

$$E(X^2) = \int_0^A t^2 g(t) dt = \int_0^A t^2 \frac{1}{M} \left[\frac{2p + (1-p) \left(\frac{x-\mu}{\beta}\right)^2}{4\beta} \right] e^{-\left|\frac{x-\mu}{\beta}\right|} dt$$

$$= \frac{1}{M} \left[\frac{p}{2\beta} \int_0^A t^2 e^{-\left|\frac{x-\mu}{\beta}\right|} dt + \frac{(1-p)}{4\beta} \int_0^A t^2 \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} dt \right]$$

Using the transformation and on simplification, one can get

$$E(X^2) = \frac{1}{M} \left[\frac{p}{2} S + \frac{(1-p)}{4} K \right]$$

Where

$$S = \beta^2 \left[e^{\frac{\mu}{\beta}} \left(\frac{\mu^2}{\beta^2} + \frac{2\mu}{\beta} + 1 \right) - e^{-\left(\frac{A-\mu}{\beta}\right)} \left(\left(\frac{A-\mu}{\beta}\right)^2 + 2 \left(\frac{A-\mu}{\beta}\right) + 1 \right) \right] + \mu^2 \left[e^{\frac{\mu}{\beta}} - e^{-\left(\frac{A-\mu}{\beta}\right)} \right]$$

$$+ 2\beta \left[e^{\frac{\mu}{\beta}} \left(1 - e^{\frac{\mu}{\beta}} \right) - e^{-\left(\frac{A-\mu}{\beta}\right)} \left(1 + \left(\frac{A-\mu}{\beta}\right) \right) \right]$$

→21

the probability density function of the complete length of service is

$$g(x) = \frac{f(x)}{F(A) - F(0)}$$

→15

Where $f(x)$ is the probability density function of given in equation (1)

$$\text{From equation (2), } F(A) = p \left[1 - \frac{e^{-\left(\frac{A-\mu}{\beta}\right)}}{2} \right] + (1-p) \left[\frac{1}{2} + \left(\frac{A-\mu}{2\beta}\right) + \left(\frac{A-\mu}{2\beta}\right)^2 \right] e^{-\left(\frac{A-\mu}{\beta}\right)}$$

$$\text{And } F(0) = \left[\frac{1}{2} + (1-p) \left(\frac{\mu^2}{4\beta^2} - \frac{\mu}{2} \right) \right] e^{-\frac{\mu}{\beta}}$$

Let $= F(A) - F(0)$, then

$$M = \left\{ p \left[1 - \frac{e^{-\left(\frac{A-\mu}{\beta}\right)}}{2} \right] + (1-p) \left[\frac{1}{2} + \left(\frac{A-\mu}{2\beta}\right) + \left(\frac{A-\mu}{2\beta}\right)^2 \right] e^{-\left(\frac{A-\mu}{\beta}\right)} \right\} - \left[\frac{1}{2} + (1-p) \left(\frac{\mu^2}{4\beta^2} - \frac{\mu}{2} \right) \right] e^{-\frac{\mu}{\beta}}$$

$$\text{Then } g(x) = \frac{1}{M} \left[\frac{2p + (1-p) \left(\frac{A-\mu}{\beta} \right)^2}{4\beta} \right] e^{-\left| \frac{x-\mu}{\beta} \right|}$$

For different values of p, μ and β the frequency curves of their distribution shown in figure - 2

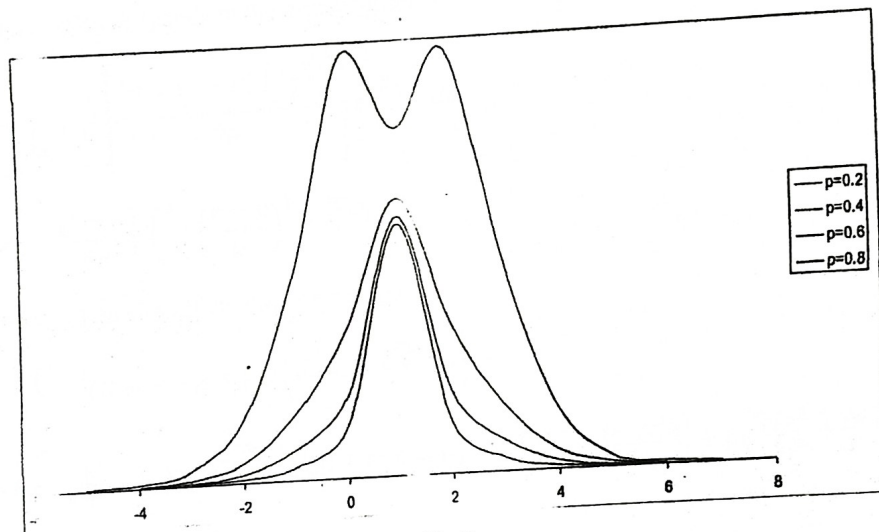


Fig 2

The probability that an employee served in an organization for the length of time 't' is $G(t)$, which is survival function of the employee in the organization is defined as the compliment of distribution function. We have distribution function of complete length of service as

$$F(t) = \int_0^t g(t) dt$$

$$= \frac{1}{M} \left[\frac{p}{2\beta} \int_0^t e^{-\left| \frac{t-\mu}{\beta} \right|} dt + \frac{(1-p)}{4\beta} \int_0^t \left(\frac{t-\mu}{\beta} \right)^2 e^{-\left| \frac{t-\mu}{\beta} \right|} dt \right]$$

$$= \frac{1}{M} \left[\frac{p}{2} \left(1 - e^{-\frac{\mu}{\beta}} \right) + \frac{(1-p)}{2} \left\{ 1 - e^{-\frac{\mu}{\beta}} \left(1 + \frac{\mu}{\beta} + \frac{\mu^2}{\beta^2} \right) \right\} \right] \text{ for } t < \mu$$

→16

$$= \frac{1}{M} \left[\frac{p}{2} \left(1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \right) + \frac{(1-p)}{2} \left\{ 1 - e^{-\left(\frac{t-\mu}{\beta}\right)} \left(1 - \left(\frac{t-\mu}{\beta}\right) + \frac{1}{2} \left(\frac{t-\mu}{\beta}\right)^2 \right) \right\} \right] \text{ for } t \geq \mu$$

→17

Since $G(t) = 1 - F(t)$,

$$G(t) = \frac{1}{M} \left[\frac{p}{2} \left(1 - e^{-\frac{\mu}{\beta}} \right) + \frac{(1-p)}{2} \left\{ 1 - e^{-\frac{\mu}{\beta}} \left(1 + \frac{\mu}{\beta} + \frac{\mu^2}{\beta^2} \right) \right\} \right] \text{ for } t < \mu$$

0.6				45		1.381
				50		1.388
0.7				58		1.4
				45		1.976
				50		2.094
0.5			3	58		2.284
				45		0.497
				50		0.342
0.6				58		0.096
				45		1.36
				50		1.367
0.7				58		1.378
				45		1.947
				50		2.065
0.5			3.1	58		2.252
				45		0.486
				50		0.333
				58		0.089
0.6				45		1.34
				50		1.347
				58		1.358

8. Conclusions

This paper is on the mixture of Laplace and two parameter Laplace type bimodal Distribution and its application on Manpower modelling. Initially the distributional properties were discussed. While applying to the Manpower modelling, two types of promotion policies namely promotion by seniority and promotion by random was considered and discussed.

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Promotion by Random

In some of the organization the promotion policy is carried out by virtue of merit considering the efficiency and expertise and not by seniority. Let $F_1(t)$ be the probability that an individual remains in the organization for time t , without setting promotion and let n_t be the promotion during $(0, t)$.
Then $F_1(t)$ = Probability (individuals not promoted in $(0, t)$ / does not leave in $(0, t)$)

$$= \text{Probability \{does not leave in } (0, t)\} = \left(1 - \frac{1}{N_1}\right)^{n_t} G(t)$$

Now the expected value of n_t is $N_1 P_t$

Then as a first approximation, one can take

$$F_1(t) = \left(1 - \frac{1}{N_1}\right)^{N_1 P_t} G(t)$$

As, $N_1 \rightarrow \infty$ gives $F_1(t) = G(t)e^{-P_t}$

Then the average time spend in grade I is

$$\theta_1 = \int_0^{\infty} F_1(t) dt = \int_0^{\infty} G(t) e^{-P_t} dt$$

→37

Using equation (28), $\frac{1}{\theta} \int_0^{\infty} G(t) e^{-P_t} dt = \frac{N_1}{N}$

Let $Q(t)$ is the probability density function of the time interval between entry of individual into grade I and his promotion to grade II given that he is promoted before leaving.

→38

$$\text{Let } Q(t) = \int_t^{\infty} q(x) dx = \left(1 - \frac{1}{N_1}\right)^{n_t}$$

Replacing n_t by its expected value $N_1 P_t$, then an approximate value of $Q(t)$ is

$$Q(t) = \left(1 - \frac{1}{N_1}\right)^{N_1 P_t}$$

→39

$$\text{Then } \mu_p = \int_0^{\infty} Q(t) dt = \int_0^{\infty} e^{-P_t} dt = \frac{1}{P}$$

Using equation (37), (38) and equation (39), one can have $\frac{N_1}{N} = \frac{1}{\theta} \int_0^A G(t) e^{\frac{1}{\mu_p} P_t} dt$

→40

$$\text{This implies that } \frac{1}{\theta} \int_0^A G(t) e^{\frac{1}{\mu_p} P_t} dt = \frac{R}{1+R}$$

For our model under consideration the $G(t)$ value is given equation (18)

$$\text{Substituting } G(t) \text{ in equation (40) gives } \frac{R}{1+R} = \frac{1}{\theta} \int_0^A G(t) e^{\frac{1}{\mu_p} P_t} dt$$

→41

By solving the (41), the explicit expression for various values of μ, β and θ the R value are computed and presented in Table :

Table 2

p	β	μ	k	A	θ	R
0.5	2	3	2.8	45	10	0.521
				50		0.362
				58		0.109
0.6				45		1.404
				50		1.412
				58		1.423
0.7				45		2.006
				50		2.126
				58		2.318
0.5			2.9	45		0.508
				50		0.352
				58		0.102

Hence $\alpha\left(\frac{t}{\theta}\right)\delta t = h(T-t)\delta t G(t)$ this hold for $t < T$

Since $\lim_{T \rightarrow \infty} h(t) = \frac{1}{\theta}$ And $\lim_{T \rightarrow \infty} G(t) = 0$, then $\lim_{T \rightarrow \infty} \alpha\left(\frac{t}{T}\right) = \frac{G(t)}{\theta} = a(t)$

Which gives the study state age distribution.

→30

In promotion by seniority, the loss form grade II is replaced by the seniority of member of grade I. It follows that at any time every individual in grade II has length of service at least as long as any individual in grade I and hence there is some thresh hold value t_1 such that all individuals with length of service less than t_1 are in grade I. t_1 is a random variable, but for large values of grade sizes the approximate formula for its expected value is given by Bartholomew, D.J. (1973) ⁽²⁾ as

$$\int_0^\infty a(t) dt = \frac{N_2}{N_1 + N_2}$$

→31

Therefore, under study state the expected number of proportion per unit time will be the promotion of new recruits whose service is at least at the threshold length t_1 . i.e.

$$N_1 p = a(t_1) \frac{N}{\theta}$$

→32

Let μ_1 be the average length of time spent in grade I by those who leave while in grade I. Let μ_p be the average length of time spent in grade I by those who are eventually promoted to grade II. Then consider the promotion problem as determining N_1 , such that μ_p is the average value of t_1 . Therefore, for equations (31) and (32), we have

$$\int_{\mu_p}^\infty a(t) dt = \frac{N_2}{N}$$

→33

$$\text{That implies } \frac{1}{\theta} \int_{\mu_p}^\infty G(t) dt = \frac{N_2}{N}$$

→34

$$\text{And } N_1 p = G(\mu_p) \frac{N}{\theta}$$

For equation (34), one can have $R = \frac{N_1}{N_2}$ as a function of μ_p and hence, knowing the total size of organization (N), one can obtain N_1 for any given value of μ_p . This implies

$$\frac{1}{\theta} \int_{\mu_p}^\infty G(t) dt = \frac{1}{1+R}$$

→35

For the model under consideration $G(t)$ is as given in equation (18) and using this in equation n (35), gives $\frac{1}{1+R} = \frac{1}{\theta} \int_{\mu_p}^\infty G(t) dt$ →36

By solving the (36), the explicit values can be obtained.

For different values of μ_p, β and p we complete the value of R and presented in Table-1

Table 1

p	β	μ_p	Λ	μ	θ	R	
0.7	2	2.8	50	3	10	0.477	
		2.9				0.466	
		3				0.456	
		2.8	58	4		0.612	
						2.9	0.601
						3	0.591
		3.8	50	4		0.642	
						3.9	0.631
						4	0.621
		3.8	58	4		0.809	
						3.9	0.799
						4	0.788
		4.8	50	5		0.72	
						4.9	0.709
						5	0.699
		4.8	58	5		0.174	
						4.9	0.163
5						0.153	
0.6		4.8	50	5		0.103	
		4.9				0.092	
		5				0.082	

$$\begin{aligned}
 & e^{\frac{t}{\beta}} \left(\frac{\mu^4}{\beta^4} - \frac{4\mu^3}{\beta^3} + \frac{12\mu^2}{\beta^2} - \frac{\mu}{\beta} 11 \right) - e^{-\left(\frac{A-\mu}{\beta}\right)} \left[\left(\frac{A-\mu}{\beta}\right)^4 + 4\left(\frac{A-\mu}{\beta}\right)^3 + 12\left(\frac{A-\mu}{\beta}\right)^2 - 24\left(\frac{A-\mu}{\beta}\right) - 1 \right] \\
 & - e^{\frac{t}{\beta}} \left(\frac{\mu^2}{\beta^2} + \frac{2\mu}{\beta} + 1 \right) - e^{-\left(\frac{A-\mu}{\beta}\right)} \left[\left(\frac{A-\mu}{\beta}\right)^2 - 2\left(\frac{A-\mu}{\beta}\right) - 1 \right] \\
 & + 2\beta e^{\frac{t}{\beta}} \left(6 - 6\frac{\mu}{\beta} + \frac{3\mu^2}{\beta^2} - \frac{\mu^3}{\beta^3} \right) - e^{-\left(\frac{A-\mu}{\beta}\right)} \left(6 + 6\left(\frac{A-\mu}{\beta}\right) + 3\left(\frac{A-\mu}{\beta}\right)^2 - \left(\frac{A-\mu}{\beta}\right)^3 \right)
 \end{aligned} \rightarrow 22$$

That is the variability of the complete length of service of an employee is

$$Var(t) = E(t^2) - [E(t)]^2 = \frac{1}{M} \left[\frac{p}{2} S + \frac{(1-p)}{4} T \right] - [E(X)]^2 \rightarrow 23$$

The renewal density of this model is

$$h(t) = g(t) + \int_0^t G(t-x)g(x)dx \rightarrow 24$$

Where $g(t)$ and $G(t)$ are as defined in (18)

Using the values of $g(t)$ and $G(t)$ in the (19) one can get the $h(t)$

A. Recruitment Policies For Manpower Model

In this section, we study the different recruitment policies of an organization, which play a vital role in Manpower Planning strategies. An organization can be viewed as having two grades. The first grade is training grade and the second grade is organization grade. The organization itself in general possess an hierarchical structure. The transfer between training grade (grade I) and organization grade (grade II) is considered as promotion. The whole organization is considered as single unit. Let N_1 and N_2 are the sizes of training grade and organization grade respectively. Then the total number of employees in the organization is $N_1 + N_2 = N$. Here N_2 is assumed to be known as fixed constant. In any organization the promotion policies are of two types namely (i) promotion by seniority (i.e.) the most senior member of the grade I is promoted to grade II and (ii) promotion by random. This is usually done based on merit. Let w_1 be the individual loss rate in training grade and w_2 is the individual loss rate in organization grade and p is the promotion rate. Since the input and output of each grade must balance, for grade II, it can be written as

$$N_1 p = N_2 w_2 \rightarrow 25$$

In equilibrium, the expected input to the organization per unit time is $\frac{N_1}{\theta}$ when θ is the mean length of completed service. Thus for grade I

$$\frac{N_1}{\theta} = N_1(p + w_1) = N_1 w_1 + N_2 w_2 \rightarrow 26$$

In general, w_1, w_2 and p are function of time. The value of p, w_1 and w_2 are independent of time, when the system is equilibrium. Let the θ_1 be the average time spent in grade I. Then under study state the expected number of vacancies arriving per unit time in this grade is

$$\frac{N_1}{\theta_1} = N_1(p + w_1) \rightarrow 27$$

$$\frac{N_1}{\theta_1} = (p + w_1) \rightarrow 28$$

$$\text{From (26) and (27)} \quad \frac{N}{\theta} = \frac{N_1}{\theta_1}$$

From this equation the throughput for the whole system is equal to the number of vacancies in the grade I.

Promotion by Seniority: Let a $\left(\frac{t}{T}\right)$ denote the age distribution of the system at time 't' give that the organization is established at time = 0. Thus, the probability that an employee chosen at random is having a length of service as T in $(t, t + \delta t)$ is $\left(\frac{t}{T}\right) \delta t$

$$\text{Therefore } \left(\frac{t}{T}\right) \delta t = \text{Probability (individual joined in } (T-t, T-t+\delta) \text{ and remaining for time } t) = h(T-t)\delta t G(t) \rightarrow 29$$

Where $h(t)$ is the renewal density of the whole System.

$$\text{As, } F(M) = 1 - F(M) \text{ and } \Rightarrow F(M) = \frac{1}{2}$$

$$\text{Therefore } \int_{-\infty}^{\infty} \left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta} \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \right] dx = \frac{1}{2}$$

$$= M = \mu$$

→5

That is for this distribution the Median = Mean and hence the distribution is said to be symmetric at $x = \mu$

Moment generating function

The moment generating function of the two-component mixture of Laplace and Laplace type variate is

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta} \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \right] dx$$

$$= \frac{p}{2\beta} \int_{-\infty}^{\infty} e^{tx} e^{-\left|\frac{x-\mu}{\beta}\right|} dx + \frac{1-p}{4\beta} \int_{-\infty}^{\infty} e^{tx} \left[\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \right] dx$$

Using the transformations and on simplification we get

$$M_X(t) = p \left[\frac{\frac{e^{t\mu}}{2}}{1 + \beta^2 t^2} \right] + (1-p) \left[\frac{e^{t\mu}}{2} \left\{ \frac{2 + 6\beta^2 t^2}{(1 - \beta^2 t^2)^3} \right\} \right]$$

$$= \frac{e^{t\mu}}{2} \left[\left(\frac{p}{1 + \beta^2 t^2} \right) + (1-p) \left\{ \frac{2 + 6\beta^2 t^2}{(1 - \beta^2 t^2)^3} \right\} \right]$$

→6

Characteristic function

The characteristic function of the variate X is

$$\Phi_X(t) = \int_{-\infty}^{\infty} e^{itx} f(x) dx = \int_{-\infty}^{\infty} e^{itx} \left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta} \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \right] dx$$

$$= \frac{p}{2\beta} \int_{-\infty}^{\infty} e^{itx} e^{-\left|\frac{x-\mu}{\beta}\right|} dx + \frac{1-p}{4\beta} \int_{-\infty}^{\infty} e^{itx} \left[\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \right] dx$$

Using the transformation, $y = x - \mu$ and on simplification one can get

$$\Phi_X(t) = p \left[\frac{\frac{e^{it\mu}}{2}}{1 + \beta^2 t^2} \right] + (1-p) \left[\frac{e^{it\mu}}{2} \left\{ \frac{2 + 6\beta^2 t^2}{(1 - \beta^2 t^2)^3} \right\} \right]$$

→7

Central Moments of the Distribution

The central moments of this distribution can be obtained as

$$\mu_{2n} = \int_{-\infty}^{\infty} (x - \mu)^{2n} f(x) dx = \int_{-\infty}^{\infty} (x - \mu)^{2n} \left[\frac{p}{2\beta} e^{-\left|\frac{x-\mu}{\beta}\right|} + \frac{1-p}{4\beta} \left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \right] dx$$

$$= \frac{p}{2\beta} \int_{-\infty}^{\infty} (x - \mu)^{2n} e^{-\left|\frac{x-\mu}{\beta}\right|} dx + \frac{1-p}{4\beta} \int_{-\infty}^{\infty} (x - \mu)^{2n} \left[\left(\frac{x-\mu}{\beta}\right)^2 e^{-\left|\frac{x-\mu}{\beta}\right|} \right] dx$$

Using the transformation and on simplification, one can get

$$\mu_{2n} = p(n)! + (1-p) \frac{(2n+2)\beta^{2n}}{2} \quad \text{If } n \text{ is even}$$

if n is odd

particular the first four central moments are

Three Parameter Laplace Type Bimodal Distribution

D V Ramana Murty¹, G Arti² and M. Vivekananda Murty³

¹Department of Statistics, VT College, Rajahmundry, East Godavari District, Andhra Pradesh, India.

²Department of Management, GITAM (Deemed to be University), Visakhapatnam, India

³Former Professor of Statistics, Andhra University, Visakhapatnam, India.

Abstract - This paper is on the Three parameter Laplace type Bimodal distribution. After discussing distributional properties, order statistics were developed and discussed. Inferential aspects were discussed and estimates of the parameters were obtained through Method of Moments and Maximum Likelihood Estimation techniques. Minimum unbiased estimator of the location parameter and best linear unbiased estimator of the location and scale parameter were also obtained.

I. INTRODUCTION

The Laplace distribution has received considerable attention as an appropriate model in reliability theory and life testing models. The statistical data in the fields like agriculture, meteorology and population studies are appearing as if it is generated from a Laplace distribution, but have the kurtosis lies between 3 and 6. In this paper we introduce a three parameter Laplace type bimodal distribution which suits the distribution arising out of the situations mentioned above. The various distributional properties and inferential aspects of this distribution are discussed.

II. THREE PARAMETER LAPLACE TYPE BIMODAL DISTRIBUTION

A random variable is said to follow a three parameter Laplace type bimodal distribution, if its probability density function is of the following form.

$$f(x, \mu, \beta) = \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \beta > 0 \quad \rightarrow 1$$

For different values of $r = 0, 1, 2, \dots, \infty$ we have different continuous distributions with three parameters μ, β and r (r can be treated as an index parameter) which can be used to determine the specific distribution. This family includes Laplace distribution, when $r = 0$ [Johnson and Kotz (1972)]. Making the transformations, $y = \frac{x-\mu}{\beta}$ in (1), we get

$$f(y) = \frac{y^{2r} e^{-|y|}}{2\beta[(2r)!]}, \quad -\infty < y < \infty \quad \rightarrow 2$$

The distribution given in (1) is symmetric about μ and it is a generalized Laplace type distribution with three parameters.

III. PROPERTIES OF THREE PARAMETER LAPLACE TYPE BIMODAL DISTRIBUTION:

The distributions belonging to this family are having the following characteristics.

The Mean of the distribution is

$$E(X) = \int_{-\infty}^{\infty} x f(x, \mu, \beta) dx = \int_{-\infty}^{\infty} x \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|} dx$$

On simplification, one can get Mean = μ

Distribution is symmetric about μ , which is the mean.

The Median of the distribution M is such that

$$\int_{-\infty}^M f(x) dx + \int_M^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^M x \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|} dx + \int_M^{\infty} x \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|} dx = 1$$

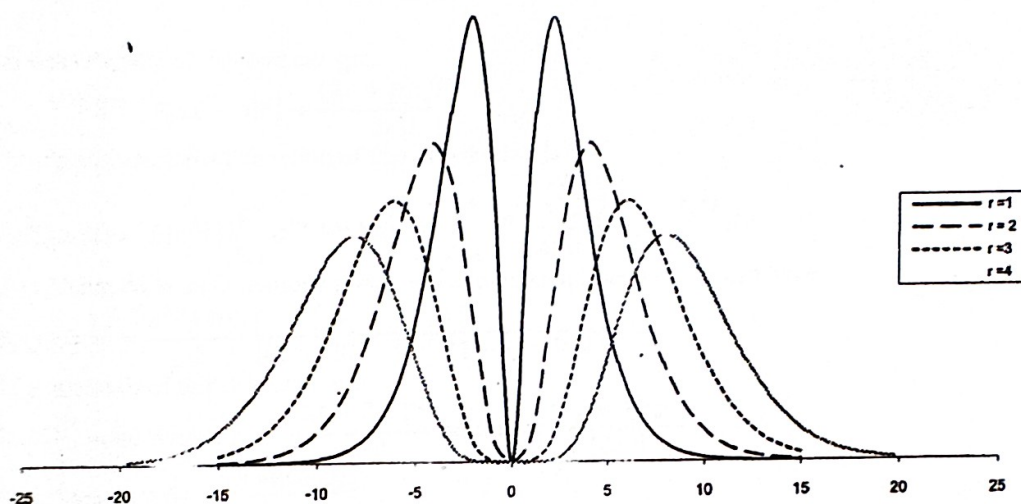
Solving the equation one can have Median = $M = \mu$

For the generalized three parameter Laplace type bimodal distribution, mean and median are equal.

For obtaining mode of the distribution, taking the derivatives of (1) with respect to x and equating to zero and solving for x , we get the modes as $\mu \pm 2r\beta$ i.e. $\mu - 2\beta$ and $\mu + 2\beta$

Therefore, this distribution is bimodal.

Fig - 4.1



Distribution function is obtained as

$$F_X(x) = \int_{-\infty}^x \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|} dx$$

On simplification, one can get

$$F_X(x) = 1 - \frac{1}{2} \sum_{i=1}^{2r} \frac{\left(\frac{x-\mu}{\beta}\right)^i}{i!} e^{-\left(\frac{x-\mu}{\beta}\right)}, \quad \text{for } x \geq \mu$$

$$= \frac{1}{2} \sum_{i=0}^{2r} \frac{\left(\frac{x-\mu}{\beta}\right)^i}{i!} e^{-\left(\frac{x-\mu}{\beta}\right)}, \quad \text{for } x < \mu \quad \rightarrow 3$$

The central moments of this distribution are

$$\mu_{2n} \int_{-\infty}^{\infty} (x-\mu)^{2n} f(x) dx = \int_{-\infty}^{\infty} (x-\mu)^{2n} \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|} dx$$

Making necessary transformation and integrating, one can get

$$\mu_{2n} = \frac{(2r+2n)! \beta^{2n}}{\beta[(2r)!]}, \mu_{2n+1} = 0 \quad \rightarrow 4$$

In particular, the first four central moments of this distribution are

$$\mu_1 = 0$$

$$\mu_2 = \frac{(2r+2)! \beta^2}{(2r)!}$$

$$\mu_3 = 0$$

$$\mu_4 = \frac{(2r+4)! \beta^4}{(2r)!} = \frac{(2r+4)(2r+3)(2r+2)! \beta^4}{(2r)!}$$

Thus, the variance of this distribution is $\mu_2 = \frac{(2r+2)! \beta^2}{(2r)!}$

Since, the distribution is symmetric, its skewness is zero.

The recurrence relation between the Central moments are

$$\mu_{2n} = \frac{(2r+2n)! \beta^{2n}}{\beta[(2r)!]} \mu_{2n-2} \quad \rightarrow 5$$

The p^{th} order absolute moments of the distribution are

$$E[|X - \mu|^p] = \int_{-\infty}^{\infty} |x - \mu|^p \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|} dx$$

On simplification, one can get,

$$E[|X - \mu|^p] = \frac{(2r+p)! \beta^p}{(2r)!} \quad \rightarrow 6$$

The characteristic function of this distribution is

$$\phi_X(t) = E[e^{itx}] = \int_{-\infty}^{\infty} e^{itx} f(x) dx = \int_{-\infty}^{\infty} e^{itx} \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|} dx$$

Using the transformations $y = x - \mu$, and by simplification one can have

$$\phi_X(t) = \frac{e^{it\mu} (2r)!}{2} \left[\frac{1}{(1+i\beta t)^{2r+1}} + \frac{1}{(1-i\beta t)^{2r+1}} \right] \quad \rightarrow 7$$

Kurtosis of this distribution is

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \text{ where } \mu_2 = \frac{(2r+2)! \beta^2}{(2r)!} \text{ and } \mu_4 = \frac{(2r+4)! \beta^4}{(2r)!}$$

$$\text{Therefore, } \beta_2 = \frac{(2r+4)(2r+3)}{(2r+2)!}$$

The distribution of the square of the variate:

Let $Y = X^2$ and $\beta = 1, \mu = 0$, then the probability density function of Y is,

$$f(y) = [(2r)!]^{-1} \sum_{i=0}^r y^{i-\frac{1}{2}} e^{-\sqrt{y}},$$

by transforming $\sqrt{y} = z, f(z) = \sum_{i=0}^r 2[(2r)!]^{-2r} z^{2i} e^{-z}$.

Which is a mixture of γ - gamma variate with parameters $z^i, i = 0, 1, 2, \dots, r$. and weights $w_i = 2(2i)! [(2r)!]^{-2r}$

IV. ORDER STATISTICS OF THREE PARAMETER LAPLACE TYPE BIMODAL DISTRIBUTION

The simple explicit form of the distribution function as given in (3) leads us to derive the Order statistics connected with this three parameter Laplace type bimodal distribution. For simplicity let us assume $\mu = 0$ and $\beta = 1$ then the probability density function (1) reduces to

$$f(x) = \frac{(x)^{2r} e^{-|x|}}{2[(2r)!]}, \quad -\infty < x < \infty \quad \rightarrow 8$$

Let $X_{1:n} \leq X_{2:n} \leq \dots \leq X_{n:n}$ denote the Order statistics obtained from a random sample of size n from the standardized Laplace type bimodal distribution having the probability density function of the form given (8).

The probability density function of m^{th} order statistics is given by

$$f_{m:n}(x) = D_{m:n} \frac{(x)^{2r} e^{-|x|}}{2[(2r)!]} \sum_{q=0}^{n-m} \binom{n-m}{q} (-1)^q \left[1 - \frac{e^{-x} (2r)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{2[(2r)!]} \right]^{m+q-1} \quad \text{for } x \geq 0$$

$$f_{m:n}(x) = D_{m:n} \frac{(x)^{2r} e^{-|x|}}{2[(2r)!]} \sum_{q=0}^{n-m} \binom{n-m}{q} (-1)^q \left[\frac{e^{-|x|} (2r)! \sum_{l=0}^{2r} \frac{x^l}{l!} e^{-|x|}}{2[(2r)!]} \right]^{m+q-1} \text{ for } x < 0$$

Where $D_{m:n} = m \binom{m}{n}$

The a^{th} moment of $X_{m:n}$ is given by

$\alpha^{(a)}_{m:n}$

$$= D_{m:n} (-1)^a \sum_{q=0}^{n-m} \binom{n-m}{q} (-1)^q \sum_{j_1 > j_2 > \dots > j_r}^{m+q-1} \binom{m+q-1}{j_1} [D(0)]^{m+q-1-j_1} \left[\prod_{l=1}^{r-1} [D(l)]^{j_l-j_{l+1}} \binom{j_l}{j_{l+1}} \right] [D(r)]^{j_r} \sum_j P_j \frac{R!}{(m+q)^{R+1}}$$

$$+ D'_{m:n} \sum_{l=0}^{m-1} \binom{m-1}{l} (-1)^l \sum_{j_1 > j_2 > \dots > j_r}^{n+l-m} \binom{n+l-m}{j_1} [D(0)]^{n+l-m-j_1} \left[\prod_{l=1}^{r-1} [D(l)]^{j_l-j_{l+1}} \binom{j_l}{j_{l+1}} \right] [D(r)]^{j_r} \sum_j P_j \frac{R!}{(n+l-m)^{R+1}}$$

Where $R = \sum_{y=1}^r \sum_{x=1}^{2j_x} j_x + 2r + a$, $D'_{m:n} = \frac{D_{m:n}}{2(2r)!}$, $D(r) = \frac{(2r)!}{2(2r)!}$

$$\text{And } P_j = \prod_{i=1}^{r-1} \binom{a}{j_{i+1}} \prod_{i=1}^{2r-1} \binom{j_{r_i}}{j_{r_i+1}} \prod_{i=1}^{2r} (i)^{-j_{r_i}} \binom{j_{2r}}{j_{r_i}} \prod_{i=1}^{2r-1} \binom{j_l}{j_{i+1}} \prod_{i=1}^{2r} (i)^{-j_l} \rightarrow 10$$

From this (10) one can calculate the expected values of Order statistics.

Distribution of Median:

To obtain the distribution of Median, substitute $m = \frac{n+1}{2}$ if n is odd in (9).

Thus, one can get

$$f_{\frac{n+1}{2}:n}(x) = D_{\frac{n+1}{2}:n} \frac{(x)^{2r} e^{-|x|}}{2[(2r)!]} \sum_{q=0}^{\frac{n-1}{2}} \binom{\frac{n-1}{2}}{q} (-1)^q \left[1 - \frac{e^{-|x|} (2r)! \sum_{l=0}^{2r} \frac{x^l}{l!} e^{-|x|}}{2[(2r)!]} \right]^{\frac{n+2q-1}{2}} \text{ for } x \geq 0$$

$$= D_{\frac{n+1}{2}:n} \frac{(x)^{2r} e^{-|x|}}{2[(2r)!]} \sum_{q=0}^{\frac{n-1}{2}} \binom{\frac{n-1}{2}}{q} (-1)^q \left[\frac{e^{-|x|} (2r)! \sum_{l=0}^{2r} \frac{x^l}{l!} e^{-|x|}}{2[(2r)!]} \right]^{\frac{n+2q-1}{2}} \text{ for } x < 0$$

Where $D_{\frac{n+1}{2}:n} = \frac{n!}{[\frac{n-1}{2}]!^2}$

when n is even then $m = \frac{2n+1}{4}$

$$f_{\frac{2n+1}{2}:n}(x) = D_{\frac{2n+1}{2}:n} \frac{x^{2r} e^{-|x|}}{2[(2r)!]} \sum_{q=0}^{\frac{2n-1}{4}} \binom{\frac{2n-1}{4}}{q} (-1)^q \left[1 - \frac{e^{-|x|} (2r)! \sum_{l=0}^{2r} \frac{x^l}{l!} e^{-|x|}}{2[(2r)!]} \right]^{\frac{2n+4q-3}{4}} \text{ for } x \geq 0$$

$$= D_{\frac{2n+1}{2}:n} \frac{x^{2r} e^{-|x|}}{2[(2r)!]} \sum_{q=0}^{\frac{2n-1}{4}} \binom{\frac{2n-1}{4}}{q} (-1)^q \left[\frac{e^{-|x|} (2r)! \sum_{l=0}^{2r} \frac{x^l}{l!} e^{-|x|}}{2[(2r)!]} \right]^{\frac{2n+4q-3}{4}} \text{ for } x < 0$$

$\rightarrow 12$

Joint Moments of the Order Statistics

The joint probability distribution of the Order Statistics $X_{m:n}$ and $X_{s:n}$, $m < s$ is given by

$$f_{m,s:n}(x,y) = D_{m,s:n} [U(x)]^{m-1} [U(y) - U(x)]^{s-m-1} [1 - U(y)]^{n-s} f(x)f(y)$$

$$\text{Where } U(x) = \frac{(2r)! e^{-|x|} \sum_{l=0}^{2r} \frac{x^l}{l!} e^{-|x|}}{2[(2r)!]} \rightarrow 13$$

Partition the range $0 < x < y < \infty$ to three mutually exclusive regions

$R_1: [(x,y): -\infty < x < y < 0]$

$R_2: [(x,y): 0 < x < y < \infty]$

$R_3: [(x,y): -\infty < x < 0, 0 < y < \infty]$

the product moments can be obtained as

$$E[X_{m,m}, X_{s,m}] = \int_{R_1 \cup R_2 \cup R_3} f(x, y) dx dy$$

$$= D_{m,s,m} \left[\sum_{l=0}^{s-m-1} \sum_{j=0}^{s-m-1} (s-m-1) \binom{n-s}{j} (-1)^{s-m-1+l+j} \Psi(s-2-l, l+j) \right.$$

$$+ \sum_{l=0}^{m-1} \sum_{j=0}^{s-m-1} (s-m-1) \binom{m-1}{j} (-1)^{l+j} \Psi(s-m-1+l+j, n-s+j) \left.
$$- \sum_{l=0}^{s-m-1} \sum_{j=0}^{s-m-1} (s-m-1-l) \binom{s-m-1}{l} (-1)^{l+j} \int_0^x x [U(x)]^{m+l-1} f(x) dx \int_0^y y [U(y)]^{n-s-j} f(y) dy \Bigg]$$$$

where

$$\Psi(a, b) = \int_0^x \int_0^y y [U(x)]^a [U(y)]^b f(x) f(y) dx dy$$

$$= \int_0^x \int_0^y (2r)^{l-2} \sum_{v=0}^{s-m-1} \sum_{u=0}^{n-s} (s-m-1) \binom{n-s}{u} (-1)^{u+v} \sum_j^{m-l+v} \sum_t^{s-m-1-v-u} P(j) Q(t) \sum_{N=0}^r \binom{r}{N}$$

$$e^{(m-1+v)x + (1-m-1-v+u)y} x^{2r+1+\sum_{l=1}^r \sum_{i=0}^{l-1} j_i} y^{2r+1+\sum_{l=1}^r \sum_{i=0}^{l-1} t_i}$$

$$P(j) = \binom{a}{j_1} \prod_{l=1}^{r-1} \left[\binom{j_l}{j_{l+1}} \right] [(2r)l]^{j_l-j_{l+1}} \binom{j_l-j_{l+1}}{j_l} [(2r)l]^{j_r} \left\{ \prod_{l=1}^r \left[\prod_{i=1}^{2l-j} \binom{j_l}{j_{l+1}} \right] \left[\prod_{i=1}^{2l} (l)^{-j_{li}} \right] \right\} \binom{j_r}{j_{rl}}$$

$$Q(t) = \binom{b}{t_1} \prod_{l=1}^{r-1} \left[\binom{t_l}{t_{l+1}} \right] [(2r)l]^{t_l-t_{l+1}} \binom{t_l-t_{l+1}}{t_l} [(2r)l]^{t_r} \left\{ \prod_{l=1}^r \left[\prod_{i=1}^{2l-1} \binom{t_{ll}}{t_{ll+1}} \right] \left[\prod_{i=1}^{2l} (l)^{-t_{li}} \right] \right\} \binom{t_r}{t_{rl}}$$

→14

V. ESTIMATION OF PARAMETERS:

Previously we have studied three parameter generalized Laplace type bimodal distribution and its distributional properties. Another aspect of any distributional study is to look in to the inferential aspects of the distribution, in particular the estimation of the parameters involved in the distribution under study. In this we will discuss the various methods of estimation by using Method of moments, maximum likelihood method of estimation, Best linear unbiased estimation in estimating the parameters of the three parameter Laplace type bimodal distribution and specially by using Numerical Analysis techniques namely Newton Raphson's method & iterative method. The asymptotic behavior of these estimators is also studied. In this distribution the value of r is fixed, based on the Kurtosis of the distribution. After identifying r we estimate these moving parameters μ and β .

A. Method of Moments:

According to this method, the moments of the population and the sample are equated correspondingly to deduce the estimators of the parameters. Let us consider a sample of size 'n' drawn from a population having the probability density function of the form given by

$$f(x, \mu, \beta) = \frac{\left(\frac{x-\mu}{\beta}\right)^{2r}}{2\beta[(2r)!]} e^{-\left|\frac{x-\mu}{\beta}\right|}$$

This distribution is having two parameters μ & β and r is identified through the sample kurtosis. Hence, we consider the first two moments of the sample and the population, which leads to the following equations.

$$\mu = \bar{x},$$

$$\frac{(2r+2)! \beta^2}{(2r)!} = s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \rightarrow 15$$

These equations give us the moment estimators as

$$\hat{\mu} = \bar{x}$$

where s^2 is the sample variance.
The variance of $\hat{\mu}$ is

$$\text{and } \hat{\beta}^2 = \frac{(2r)! s^2}{(2r+2)!}$$

$$\text{Var}(\bar{x}) = \text{Var}\left(\frac{1}{n} \sum_{i=1}^n x_i\right) = \frac{1}{n^2} \sum_{i=1}^n \text{Var}(x_i) = \frac{1}{n^2} \sum_{i=1}^n \frac{(2r)! s^2 \beta^2}{(2r+2)!} = \frac{\beta^2 (2r+2)!}{n (2r)!} \rightarrow 16$$

and $\frac{1}{n} \sum_{i=1}^n E(x_i) = \mu$
That is the sample mean is an unbiased estimator of $\mu \rightarrow 17$

The unbiased estimator of β^2 is $\hat{\beta}^2 = \frac{(2r)! s^2}{(2r+2)!}$
where s^2 is the sample variance.

$\rightarrow 18$

The variance of the estimators are given by

$$\text{Var}(\hat{\mu}) = \frac{\beta^2 (2r+2)!}{n (2r)!}$$

$\rightarrow 19$

and the unbiased estimator of β^2 ,

$$\text{Var}(\hat{\beta}^2) = \beta^4 \left[\frac{(2r)!}{(2r+2)!} \right]^2 \left\{ \left[\frac{(2r+4)!}{(2r)!} \right] \left(\frac{1}{n} - \frac{2}{n^2} - \frac{1}{n^3} \right) - \left[\frac{(2r+4)!}{(2r)!} \right]^2 \left[\frac{1}{n} - \frac{4}{n^2} - \frac{3}{n^3} \right] \right\} \rightarrow 20$$

As the variance tends to zero when n tends to ∞ , the estimators $\bar{x}$ and $\hat{\beta}^2$ are consistent for μ and β^2 . Similar to the simple Laplace distribution the median is the maximum likelihood estimators of the parameter μ . Hence estimation of all the parameters can be done sequentially with μ being estimated by the median and then the other two parameters can be simultaneously estimated conditioned to $\mu = \hat{\mu}$.

B. Maximum Likelihood Method of Estimation:

Let $x_1, x_2, \dots, x_n$ be a sample of size n drawn from a population having the probability density function of the form given in equation (1). From equation (1) one can write the likelihood function of the sample is

$$L = \frac{\prod_{i=1}^n \left[\left(\frac{x_i - \mu}{\beta} \right)^{2r} \right] e^{-\frac{|x_i - \mu|}{\beta}}}{2\beta [(2r)!]} \rightarrow 21$$

Taking logarithms on both sides of (4.6.1) we get

$$\log L = n + \frac{2r}{\beta} \sum_{i=1}^n \frac{(x_i - \mu)}{\left(\frac{x_i - \mu}{\beta} \right)^2} - \frac{1}{\beta} \sum_{i=1}^n |x_i - \mu| = 0 \rightarrow 22$$

For obtaining the maximum likelihood estimators of the parameters we have to maximize L or $\log L$ with respect to the parameters μ and β . The values of μ and β can be obtained by solving the following equations.

$$\frac{(2r)!}{(2r+2)!} - r \sum_{i=1}^n \frac{1}{\left(\frac{x_i - \mu}{\beta} \right)^2} = 0$$

$$\text{That is } \sum_{i=1}^n \frac{1}{\left(\frac{x_i - \mu}{\beta} \right)^2} - 2\beta [(2r)!] = 0 \rightarrow 23$$

C. Best Linear Unbiased Estimators:

The best linear unbiased estimators of the location and scale parameters involved in the three parameter Laplace type bimodal distribution having the probability density function of the form given in (1) are obtained as follows. For that consider the family of distributions given in equation (1)

Let $X_{m:n}$ and $X_{m:n}$ be the m^{th} order statistics drawn from the populations having the probability density function of the form given above and the corresponding standardized distribution respectively.

$$\text{i.e. } X_{m:n} = \frac{X_{m:n} - \mu}{\beta}, \quad 1 \leq m \leq n$$

Then the best linear unbiased estimators of μ and β are given by [Lloyd (1952)].

$$\mu^* = \frac{l'\Omega X}{l'\Omega l} \text{ and } \beta^* = \frac{\alpha'\Omega X}{\alpha'\Omega \alpha}$$

where $l' = (1, 1, \dots, 1)$, $X' = [X_{1:n}, X_{2:n}, \dots, X_{n:n}]$ and $\alpha' = [\alpha_{1:n}, \alpha_{2:n}, \dots, \alpha_{n:n}]$

where, $\alpha_{1:n}$ is the first moment of their i^{th} order statistics and Ω is the variance matrix of vector X

The variance of these estimators are $\text{var}(\mu^*) = \frac{\sigma}{l'\Omega l}$, $\text{var}(\beta^*) = \frac{\sigma}{\alpha'\Omega \alpha}$ and $\text{cov}(\mu^*, \beta^*) = 0$

With the moments of the order statistics given in section 4 and 5, one can compute the μ^* and β^* and their variances respectively.

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Two Parameter Laplace Type Bimodal Distribution

D V Ramana Murty*, G Arti** and M. VivekanandaMurty***

*Department of Statistics, VT College, Rajahmundry, East Godavari District, Andhra Pradesh, India.

** Department of Management, GITAM (Deemed to be University), Visakhapatnam, India.

and ***Former Professor of Statistics, Andhra University, Visakhapatnam, India.

Abstract

This paper is on the two parameter Laplace type Bimodal distribution. After discussing distributional properties, order statistics were developed and discussed. Inferential aspects were discussed and estimates of the parameters were obtained through Method of Moments and Maximum Likelihood Estimation techniques. Minimum unbiased estimator of the location parameter and best linear unbiased estimator of the location and scale parameter were also obtained.

Key words: Two parameter Laplace, Bimodal, Order statistics, Estimation of Parameters

I. INTRODUCTION

The Laplace distribution has received considerable attention as an appropriate model in reliability theory and life testing models. The statistical data in the fields like agriculture, meteorology and population studies are appearing as if it is generated from a Laplace distribution, but have the kurtosis lies between 3 and 6. In this paper we introduce a two parameter Laplace type bimodal distribution which suits the distribution arising out of the situations mentioned above. The various distributional properties and inferential aspects of this distribution are discussed. This distribution can be viewed as a reflected gamma type distribution and a hybridization of Laplace and reflected two parameter gamma distribution.

II. TWO PARAMETER LAPLACE TYPE BIMODAL DISTRIBUTION

A continuous random variable X is said to have a two parameter Laplace type bimodal distribution if its probability density function is of the form,

$$f(x, \mu, \beta) = \frac{1}{4\beta} \left(\frac{x - \mu}{\beta} \right)^2 e^{-\left| \frac{x - \mu}{\beta} \right|}, \quad -\infty < x < \infty, \quad \infty < \mu < \infty, \quad \beta > 0 \quad \rightarrow 1$$

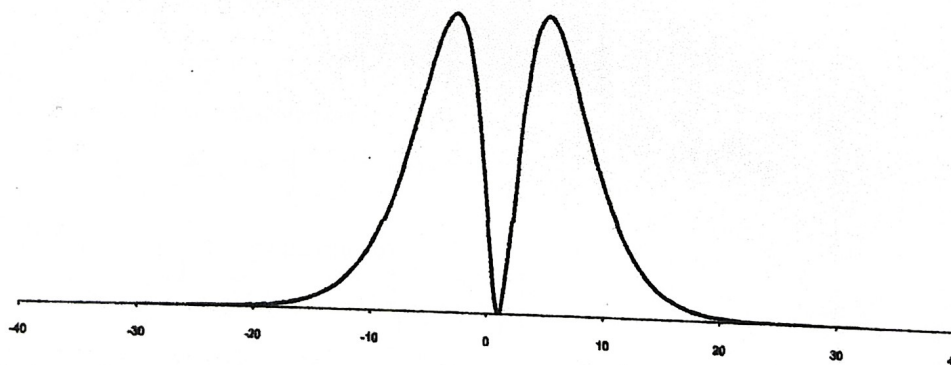
Making the transformations, $y = \frac{x - \mu}{\beta}$ in equation (1), one can get

$$f(y) = \frac{1}{4} y^2 e^{-|y|}, \quad -\infty < y < \infty \quad \rightarrow 2$$

which can be called as the standard Laplace type bimodal distribution.

The various shapes of the frequency curves are shown in figure 1.

Case 1: $\mu = 1, \beta = 2$



$$\alpha_{m:n}(x) =$$

$$\int_0^{\infty} (-x)^a D_{m:n} x^{n-m} e^{-x} \sum_{q=0}^{n-m} \binom{n-m}{q} (-1)^q \left[\frac{\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^{m+q-1} dx +$$

$$\int_0^{\infty} (x)^a D_{m:n} x^2 e^{-x} \sum_{q=0}^{n-m} \binom{n-m}{q} (-1)^q \left[1 - \frac{\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^{m+q-1} dx \rightarrow 19$$

Define $D_{m:n} = \frac{n!}{m!}$ and after simplifying $\alpha_{m:n}(a)$

$$= (-1)^a D_{m:n} \sum_{k=0}^l \sum_{q=0}^{n-m} \sum_{j=0}^{m+q-1} \sum_{l=0}^j \sum_{p=0}^l \binom{l}{k} \binom{n-m}{q} \binom{m+q-1}{j} \binom{j}{l} \binom{l}{p} 2^{j-1+l-p} (-1)^q \frac{(a+2k+l+p)!}{(m+q)^{a+2k+l+p+1}}$$

$$+ D_{m:n} \sum_{k=0}^l \sum_{q=0}^{n-m} \sum_{j=0}^{m+q-1} \sum_{l=0}^j \sum_{p=0}^l \binom{l}{k} \binom{n-m}{q} \binom{m+q-1}{j} \binom{j}{l} \binom{l}{p} 2^{j-l-p} \frac{(a+2k+l+p)!}{(1+l)^{a+2k+l+p+1}} \rightarrow 20$$

Probability distribution of first ordered statistics is

$$f_{1:n}(x) = \frac{n}{4} x^2 e^{-x} \sum_{q=0}^{n-1} \binom{n-1}{q} (-1)^q \left[\frac{1 - \sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-x}}{4} \right]^q \text{ for } x \geq 0$$

$$f_{1:n}(x) = \frac{n}{4} x^2 e^{-|x|} \sum_{q=0}^{n-1} \binom{n-1}{q} (-1)^q \left[\frac{\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^q \text{ for } x < 0 \rightarrow 21$$

The expected value of the first order statistics is

$$E[X_{(1)}] = n \left[- \sum_{k=0}^l \sum_{q=0}^{n-m} \sum_{j=0}^{m+q-1} \sum_{l=0}^j \sum_{p=0}^l \binom{l}{k} \binom{n-1}{q} \binom{q}{j} \binom{j}{l} \binom{l}{p} 2^{l-p} \frac{(a+2k+1+p)!}{(1+q)^{a+2k+l+p+1}} \right]$$

$$+ \sum_{k=0}^l \sum_{q=0}^{n-1} \sum_{l=0}^{m+q-1} \sum_{i=0}^l \sum_{p=0}^i \binom{l}{k} \binom{n-1}{q} \binom{q}{l} \binom{l}{i} \binom{i}{p} 2^{l-p} \frac{(a+2k+i+p)!}{(1+l)^{a+2k+l+p+1}} \rightarrow 22$$

The n th order statistics of the distribution is same as the first order statistic with negative variate because the distribution is symmetric.

A. Distribution of the Median

To obtain the distribution of the median, substitute $m = \frac{n+1}{2}$ if n is odd in (17).

Then one can get

$$f_{\frac{n+1}{2}:n}(x) = D_{\frac{n+1}{2}:n} \frac{1}{4} x^2 e^{-|x|} \sum_{q=0}^{\frac{n-1}{2}} \binom{\frac{n-1}{2}}{q} (-1)^q \left[\frac{1 - \sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^{\frac{n+2q-1}{2}},$$

$$\text{for } x \geq 0$$

$$= D_{\frac{n+1}{2}:n} \frac{1}{4} x^2 e^{-|x|} \sum_{q=0}^{\frac{n-1}{2}} \binom{\frac{n-1}{2}}{q} (-1)^q \left[\frac{\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^{\frac{n+2q-1}{2}} \text{ for } x < 0 \rightarrow 23$$

$$\text{Where, } D_{\frac{n+1}{2}:n} = \frac{n!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}$$

When n is even, the distribution of the median is

sample and to determine the linear coefficients $l_1, l_2, \dots, l_n$ so that the estimator is unbiased and minimum variance in this class of estimators. [C.R.Rao (1974)]. Here note that the random sample here means a set of i. i. d random variables.

The condition of unbiasedness leads to $l_1 + l_2 + \dots + l_n = 1$ and the condition of minimum variance leads to $l_1^2 + l_2^2 + \dots + l_n^2$ being minimum.

Writing, $\hat{l} = \frac{l_1 + l_2 + \dots + l_n}{n}$ one can have $\sum_{i=1}^n (l_i - \bar{l})^2 \geq 0$

and this implies, $\sum_{i=1}^n l_i^2 \geq n\bar{l}^2 = \frac{1}{n}$, since $\sum_{i=1}^n l_i = 1$

Equality occurs only when all the l_i 's are equal, i.e. $l_i = \frac{1}{n}$

Thus, the greatest lower bound of $\sum_{i=1}^n l_i^2$ for various l_i is $\frac{1}{n}$ and is attained.

Therefore, $\hat{\mu} = \bar{x}$ is the minimum variance unbiased estimator. The same holds for any value of μ . Hence it is also uniformly minimum variance unbiased estimator.

The minimum variance in this case is $\frac{\text{var}(\bar{x})}{n} = \frac{12\beta^2}{n}$

For obtaining the minimum variance unbiased estimator of β^2 , consider the usual estimator $\hat{\beta}^2$ which is unbiased. It can also be observed that it is the minimum variance unbiased estimator of β^2

D. Best Linear Unbiased Estimators

Following the heuristic argument of Govindarajulu (1966) the best linear unbiased estimators of the location, and scale parameters involved in the two parameter Laplace type bimodal distribution having the probability density function of the form given in (1) are obtained as follows. Then the best linear unbiased estimators of μ and β are given by Lloyd (1952) as

$$\mu^* = \frac{l' \Omega X}{l' \Omega l} \text{ and } \beta^* = \frac{\alpha' \Omega X}{\alpha' \Omega \alpha}$$

where $l' = (1, 1, \dots, 1)$, $X' = [X_{1:n}, X_{2:n}, \dots, X_{n:n}]$ and $\alpha' = [\alpha_{1:n}, \alpha_{2:n}, \dots, \alpha_{n:n}]$ and $\alpha_{1:n}$ is the first moment of the i^{th} order statistics and Ω is the variance matrix of vector X

The variance of these estimators are

$$\text{var}(\mu^*) = \frac{12\beta^2}{l' \Omega l}, \text{var}(\beta^*) = \frac{12\hat{\beta}^2}{\alpha' \Omega \alpha}$$

Where Ω is the standard deviation of the population and $\text{cov}(\mu^*, \beta^*) = 0$. With the moments of the order statistics given in section (2.4) one can compute μ^* and β^* and their variances respectively.

VI. CONCLUSIONS

This paper is on the distributional properties of the two parameter Laplace type bimodal distribution. The order statistic was also studied. In addition, inferential aspects like estimation of the parameters by method of moments, Maximum likelihood method of estimation were explained. Minimum variance unbiased estimators and Best linear unbiased estimators for the parameters of the distribution were presented.

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$$\text{and } \Psi(a, b) = \int_0^x \int_0^y xy [U(x)]^a [U(y)]^b f(x)f(y) dx dy \quad \rightarrow 27$$

$$\begin{aligned} \Psi(a, b) &= \int_0^x \int_0^y xy \left[\frac{\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^a \left[\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{y^i}{i!} e^{-|y|} \right]^b \left[\frac{2+x^2}{4} \right] e^{-x} \left[\frac{2+y^2}{4} \right] e^{-y} dx dy \\ &= \frac{1}{4^{a+b+2}} \sum_{k=0}^l \binom{l}{k} \sum_{j=0}^a \binom{a}{j} \sum_{i=0}^r \binom{r}{i} \sum_{i=0}^j \binom{j}{i} 2^{r-i} \left[\frac{\Gamma(2k+j+i+2)}{(r+1)(2k+j+i+3)} \right. \\ &\quad \left. - \sum_{j_1=0}^{2k+j+i+2} \frac{(r+1)^{j_1} \Gamma(2k+j+j_1+i+2)}{j_1! [2(r+1)]^{2k+j+j_1+i+3}} \right] \rightarrow 28 \end{aligned}$$

Substituting the value of $\Psi(a, b)$ of (28) in (20), one can get the moments of the order statistics.

V. INFERENCE ASPECTS OF THE TWO PARAMETER LAPLACE TYPE BIMODAL DISTRIBUTION

In the earlier section the two parameter Laplace type bimodal distribution was studied its distributional properties were presented. Another aspect of any distributional study is to look into the inferential aspects of the distribution, in particular the estimation of the parameters involved in the distribution under study. In this section various methods of estimation using the method of moments, maximum likelihood method estimation and best linear unbiased estimation in estimating the parameters of the two parameter Laplace type bimodal distribution were discussed. The asymptotic behaviors of these estimators are also studied.

A. Method of moments

Let us consider a sample of size 'n' drawn from a population having the probability density function of the form given by

$$f(x, \mu, \beta) = \frac{1}{4\beta} \left(\frac{x-\mu}{\beta} \right)^2 e^{-\frac{|x-\mu|}{\beta}}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \beta > 0$$

This distribution is having two parameters μ and β . Hence, we consider the first two moments of the sample and the population, which leads to the following equations.

$$\mu = \bar{x}, \quad 12\beta^2 = s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

These equations give us the moment estimators as

$$\hat{\mu} = \bar{x} \text{ and } \hat{\beta}^2 = \frac{s^2}{12} = 0.0833s^2 \quad \rightarrow 29$$

The variance of $\hat{\mu}$ is

$$\text{Var}(\bar{x}) = \text{Var}\left(\frac{1}{n} \sum_{i=1}^n x_i\right) = \frac{1}{n^2} \sum_{i=1}^n \text{Var}(x_i) = \frac{1}{n^2} (12\beta^2) \quad \rightarrow 30$$

$$\text{and } E(\bar{x}) = E\left(\frac{1}{n} \sum_{i=1}^n x_i\right) = \frac{1}{n} \sum_{i=1}^n E(x_i) = \mu$$

Therefore, the sample mean is an unbiased estimator of μ .

For obtaining the unbiased estimator of β^2 , consider

$$\hat{\beta}^{*2} = \frac{n}{n-1} (0.0833)s^2$$

$$E(\hat{\beta}^{*2}) = \beta^2$$

Therefore, $\hat{\beta}^{*2}$ is an unbiased estimator of β^2 .

The variance of $\hat{\beta}^{*2}$ can be obtained as

$$\text{Var}(\hat{\beta}^{*2}) = \text{Var}\left[\frac{n}{n-1} (0.0833)s^2\right] = \frac{n^2}{(n-1)^2} (0.0833)^2 \text{Var}(s^2) \quad \rightarrow 32$$

Where $Var(s^2) = \frac{\mu_4 - \mu_2^2}{n} - \frac{2(\mu_4 - \mu_2^2)}{n^2} + \frac{(\mu_4 - 3\mu_2^2)}{n^3}$
and μ_i is the i^{th} central moment (Cramer, 1946)

Therefore,

$$Var(s^2) = \frac{(360 - 144) \beta^4}{n} - \frac{2(360 - 288) \beta^4}{n^2} + \frac{(360 - 432) \beta^4}{n^3} = \left[\frac{216}{n} - \frac{144}{n^2} - \frac{72}{n^3} \right] \beta^4 \rightarrow 33$$

Equations (32) and (33) gives

$$Var(\hat{\beta}^{*2}) = \frac{n^2}{(n-1)^2} \left[\frac{216}{n} - \frac{144}{n^2} - \frac{72}{n^3} \right] (0.0833)^2 \beta^4 \rightarrow 34$$

From the equations (32) and (34) it can be observed that as $n \rightarrow \infty$ the variances tend to zero. Hence the estimators $\bar{x}$ and $\hat{\beta}^{*2}$ are consistent. Since the sample observations are independently distributed, these estimators are strongly consistent. Since the covariance of $\bar{x}$ and $\hat{\beta}^{*2}$ is zero, these estimators $\bar{x}$ and $\hat{\beta}^{*2}$ are uncorrelated. This result can also be visualized from the fact that the sample is drawn from a symmetric distribution.

B. Maximum Likelihood Method of Estimation

Let $x_1, x_2, \dots, x_n$ be a sample of size n drawn from a population having the probability density function of the form given in equation (1). Then the likelihood function of the sample is

$$L = \frac{1}{(4\beta)^n} \prod_{i=1}^n \left[\frac{|x_i - \mu|}{\beta} \right] e^{-\sum_{i=1}^n \frac{|x_i - \mu|}{\beta}} \rightarrow 35$$

Taking logarithms on both sides of (35) gives

$$\log L = -n \log(4\beta) + \sum_{i=1}^n \log \left(\frac{|x_i - \mu|}{\beta} \right) - \sum_{i=1}^n \frac{|x_i - \mu|}{\beta} \rightarrow 36$$

For obtaining the maximum likelihood estimators of the parameters one has to maximize L or $\log L$ with respect to the parameters μ and β . Since equation (32) is not differentiable with respect to μ , the likelihood estimator of μ can be obtained as follows.

Since the density function is symmetric about $x = \mu$, i.e., $f(x + \mu) = f(x - \mu)$ it follows that $E(x) = \mu$. Hence $\frac{1}{n} \sum_{i=1}^n |x_i - \mu|$ is the mean deviation of the random sample from its mean. This shall be minimum if μ is the median of the sample. Since

$$\text{Max}(\log L) = \text{Max} \left\{ \sum_{i=1}^n \log \left(\frac{|x_i - \mu|}{\beta} \right) - \sum_{i=1}^n \frac{|x_i - \mu|}{\beta} \right\} = \text{Min} \sum_{i=1}^n |x_i - \mu|$$

it can be shown that $\hat{\mu} = \text{Median of the sample}$ if n is odd.
If n is even, then $X_{\frac{n}{2}-1} \leq \hat{\mu} \leq X_{\frac{n}{2}+1}$, where X_j is the j^{th} order statistic.

After obtaining the maximum likelihood estimation of μ as $\hat{\mu}$ and substituting $\hat{\mu}$ in equation (32) the maximum likelihood estimation of β^2 can be obtained by differentiating $\log L$ with respect to β^2 and equating it to zero. After simplification one can obtain

$$\hat{\beta}^2 = \frac{-n + \sum_{i=1}^n |x_i - \hat{\mu}|}{2n}$$

$$\text{Also, since } E \left[\frac{-\partial \log L}{\partial \beta^2} \right]^{-1} = E \left[\frac{n}{2\beta^4} + \frac{1}{2} \frac{\sum_{i=1}^n |x_i - \mu|}{\beta^4} \right]^{-1} = \frac{2\beta^4}{n}$$

The approximate expression to the asymptotic variance of $\hat{\beta}^2$ can be obtained as

$$\text{Asyvar}(\hat{\beta}^2) = E \left[\frac{-\partial^2 \log L}{\partial (\beta^2)^2} \right]^{-1} = \frac{2\beta^4}{n}$$

C. Minimum variance unbiased estimators

In this method the minimum variance unbiased estimators of the location parameter β^2 can be obtained. The general method of estimating μ is to consider the linear combination $l_1 x_1 + l_2 x_2 + \dots + l_n x_n$ of the random

$$f_{\frac{2n+1}{4}}(x) = D_{\frac{2n+1}{4},n} \frac{1}{4} x^2 e^{-|x|} \sum_{q=0}^{\frac{2n-1}{4}} \left(\frac{2n-1}{4} \right) (-1)^q \left[\frac{1 - \sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^{\frac{2n+2q-1}{4}}$$

$$= D_{\frac{2n+1}{4},n} \frac{1}{4} x^2 e^{-|x|} \sum_{q=0}^{\frac{2n-1}{4}} \left(\frac{2n-1}{4} \right) (-1)^q \left[\frac{\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^{\frac{2n+4q-3}{4}}$$

for $x \geq 0$

for $x < 0 \rightarrow 24$

Where $D_{\frac{2n+1}{4},n} = \frac{n!}{\left(\frac{2n-3}{4}\right)!^2}$

B. Joint Moments of the Order Statistics

The joint probability density function of the m^{th} and s^{th} order statistics $X_{m:n}$ and $X_{s:n}$, $m < s$ is given by [David (1981)]

$$f_{m,s:n}(x) = D_{m,s:n} [F(x)]^{m-1} [F(y) - F(x)]^{s-m-1} [1 - F(y)]^{n-s} f(x) f(y)$$

Where $D_{m,s:n} = \frac{n!}{(m-1)!(s-m-1)!(n-s)!}$

And $F(x)$ is the cumulative density function of the laplace type distribution as given in equation (3)

Writing $(x) = \frac{2+|x|+x^2}{1}$, it is possible to express the joint density function of $X_{m:n}$ and $X_{s:n}$ as

$$f_{m,s:n}(x) = D_{m,s:n} [U(x)]^{m-1} [U(y) - U(x)]^{s-m-1} [1 - U(y)]^{n-s} f(x) f(y) \rightarrow 25$$

where $D_{m,s:n} = \frac{n!}{(m-1)!(s-m-1)!(n-s)!}$ for $[(x,y): -\infty < x < y < \infty]$

This region can be split into three mutually exclusive regions,

$$R_1: [(x,y): -\infty < x < y < 0]$$

$$R_2: [(x,y): 0 < x < y < \infty]$$

$$R_3: [(x,y): -\infty < x < \infty, -\infty < y < \infty]$$

With this splitting of the region of the product moments can be obtained as

$$E[X_{m:n}, X_{s:n}] =$$

$$D_{m,s:n} \left\{ \iint_{R_1} xy [U(-x)]^{m-1} [U(-y) - U(-x)]^{s-m-1} [1 - U(-y)]^{n-s} f(-x) f(-y) dx dy \right. \\ \left. + \iint_{R_2} xy [1 - U(x)]^{m-1} [U(x) - U(y)]^{s-m-1} [1 - U(y)]^{n-s} f(x) f(y) dx dy \right. \\ \left. - D_{m,s:n} \left\{ \iint_{R_3} (-xy) [U(x)]^{m-1} [1 - U(-x) - U(-y)]^{s-m-1} [U(y)]^{n-s} f(x) f(y) dx dy \right\} \rightarrow 26 \right.$$

Expanding the binomial expression in the integral, one can have

$$E[X_{m:n}, X_{s:n}] = D_{m,s:n} \left[\sum_{i=0}^{s-m-1} \sum_{j=0}^{n-s} \binom{s-m-1}{i} \binom{n-s}{j} (-1)^{s-m-1+i+j} \Psi(s-2-i, i+j) \right] \\ + \iint_{R_2} xy [1 - U(x)]^{m-1} [U(x) - U(y)]^{s-m-1} [1 - U(y)]^{n-s} f(x) f(y) dx dy \\ - D_{m,s:n} \left[\sum_{i=0}^{s-m-1} \sum_{j=0}^{n-s} \binom{s-m-1}{j} \binom{n-s}{j} (-1)^{i+j} \Psi(s-m-1+i+j, n-s+j) \right] \\ - D_{m,s:n} \left[\sum_{i=0}^{s-m-1} \sum_{j=0}^{n-s} \binom{s-m-1}{j} \binom{n-s}{j} (-1)^{i+j} \int_0^x x [U(x)]^{m+i-1} f(x) dx \int_0^\infty y [U(y)]^{n-s-j} f(y) dy \right]$$

where $D_{m,s:n}$ is given in (25)

$$f(x) = \frac{1}{4\beta} \left(\frac{x}{\beta}\right)^2 e^{-\frac{|x|}{\beta}}, \quad -\infty < x < \infty \quad \rightarrow 10$$

The sample observations $x_1, x_2, \dots, x_n$ are independently identically distributed as given in (10). Since $x_1, x_2, \dots, x_n$ are independently distributed, the characteristic function of $x_1, x_2, \dots, x_n$ is given by

$$\phi(t) = \frac{1}{4\beta} \int_{-\infty}^{\infty} e^{it|x|} \left(\frac{x}{\beta}\right)^2 e^{-\frac{|x|}{\beta}} dx \quad \rightarrow 11$$

If $u = \sum_{i=1}^n x_i$ then the distribution function of, namely $p(u)$, is given by

$$p(u) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{itu} \left\{ \frac{1}{4\beta} \int_{-\infty}^{\infty} e^{it|x|} \left(\frac{x}{\beta}\right)^2 e^{-\frac{|x|}{\beta}} dx \right\}^n du \quad \rightarrow 12$$

On simplification

$$p(u) = \frac{1}{2\pi} \frac{1}{2^n \beta^n} \int_{-\infty}^{\infty} e^{itu} \frac{1}{(1 - i\beta t)^{3n}} du \quad \rightarrow 13$$

The poles of the integrand are of the $3n^{\text{th}}$ order and one of those is $(1 - i\beta t)^{3n}$. From the well-known residue theorem of Cauchy Macro Bert (1933), one can have

$$p(u) = \frac{1}{2^n \beta^n} \frac{1}{i^{3n-1}} \frac{d^{3n-1}}{dt^{3n-1}} \left[\frac{e^{-i\mu t}}{(1 - i\beta t)^{3n}} \right] \Bigg|_{t=\frac{1}{\beta i}} \quad \rightarrow 14$$

The distribution of arithmetic mean of the sample mean of the sample size n is

$$p(\bar{x}) = \frac{n}{3^n \beta^n} \sum_{j=1}^n \frac{(-1)^{n+2j-1}}{(n+2j-i)! i(n+2j-1)} \frac{1}{dt^{n+2j-1}} \left[\frac{e^{-itn|\bar{x}|}}{(1+i\beta t)^{n+2j}} \right] \Bigg|_{t=\frac{1}{\beta i}} \quad \rightarrow 15$$

defined for all values of on the range $(-\infty < \bar{x} < \infty)$.

IV. ORDER STATISTICS OF TWO PARAMETER LAPLACE TYPE BIMODAL DISTRIBUTION

The simple explicit form of the distribution function as given in (3) leads to derive the order statistics connected with this two parameter Laplace type bimodal distribution. For simplicity, let us assume $\mu = 0, \beta = 1$ then the probability density function (1) reduces to

$$f(x) = \frac{1}{4} x^2 e^{-|x|}, \quad -\infty < x < \infty \quad \rightarrow 16$$

Let $X_{1:n} \leq X_{2:n} \leq \dots \leq X_{n:n}$ denote the order statistics obtained from a random sample of size 'n' from the standardized Laplace distribution having the probability density function of the form given in (1).

The probability density function of the m^{th} order statistics is given by [David (1991)].

$$f_{m:n}(x) = D_{m:n} [f(x)]^{m-1} [1 - F(x)]^{n-m} f(x), \quad -\infty < x < \infty$$

where

$$D_{m:n} = \frac{n!}{(m-1)!(n-m)!} \quad \rightarrow 17$$

Substituting $F(x)$ values in the above equation (17), one can get

$$f_{m:n}(x) = D_{m:n} \frac{1}{4} x^2 e^{-|x|} \sum_{q=0}^{n-m} \binom{n-m}{q} (-1)^q \left[1 - \sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|} \right]^{m+q-1} \quad \text{for } x \geq 0$$

$$f_{m:n}(x) = D_{m:n} \frac{1}{4} x^2 e^{-|x|} \sum_{q=0}^{n-m} \binom{n-m}{q} (-1)^q \left[\frac{\sum_{k=0}^l \binom{l}{k} (2k)! \sum_{i=0}^{2k} \frac{x^i}{i!} e^{-|x|}}{4} \right]^{m+q-1} \quad \text{for } x < 0$$

$\rightarrow (18)$

where, $D_{m:n}$ is as given in (17).

The a^{th} moment of $X_{m:n}$ is given by

Where $Var(s^2) = \frac{\mu_4 - \mu_2^2}{n} - \frac{2(\mu_4 - \mu_2^2)}{n^2} + \frac{(\mu_4 - 3\mu_2^2)}{n^3}$
and μ_i is the i^{th} central moment (Cramer, 1946)

Therefore,

$$Var(s^2) = \frac{(360 - 144) \beta^4}{n} - \frac{2(360 - 288) \beta^4}{n^2} + \frac{(360 - 432) \beta^4}{n^3} = \left[\frac{216}{n} - \frac{144}{n^2} - \frac{72}{n^3} \right] \beta^4 \rightarrow 33$$

Equations (32) and (33) gives

$$Var(\hat{\beta}^2) = \frac{n^2}{(n-1)^2} \left[\frac{216}{n} - \frac{144}{n^2} - \frac{72}{n^3} \right] (0.0833)^2 \beta^4 \rightarrow 34$$

From the equations (32) and (34) it can be observed that as $n \rightarrow \infty$ the variances tend to zero. Hence the estimators $\bar{X}$ and $\hat{\beta}^2$ are consistent. Since the sample observations are independently distributed, these estimators are strongly consistent. Since the covariance of $\bar{X}$ and $\hat{\beta}^2$ is zero, these estimators $\bar{X}$ and $\hat{\beta}^2$ are uncorrelated. This result can also be visualized from the fact that the sample is drawn from a symmetric distribution.

B. Maximum Likelihood Method of Estimation

Let $x_1, x_2, \dots, x_n$ be a sample of size n drawn from a population having the probability density function of the form given in equation (1). Then the likelihood function of the sample is

$$L = \frac{1}{(4\beta)^n} \prod_{i=1}^n \left[\left(\frac{x_i - \mu}{\beta} \right)^2 \right] e^{-\sum_{i=1}^n \left(\frac{x_i - \mu}{\beta} \right)^2} \rightarrow 35$$

Taking logarithm on both sides of (35) gives

$$\log L = -n \log(4\beta) + \sum_{i=1}^n \log \left(\frac{x_i - \mu}{\beta} \right)^2 - \sum_{i=1}^n \left| \frac{x_i - \mu}{\beta} \right| \rightarrow 36$$

For obtaining the maximum likelihood estimators of the parameters one has to maximize L or $\log L$ with respect to the parameters μ and β . Since equation (32) is not differentiable with respect to μ , the likelihood estimator of μ can be obtained as follows.

Since the density function is symmetric about $x = \mu$, $e f(x + \mu) = f(x - \mu)$ it follows that $E(x) = \mu$. Hence $\frac{1}{n} \sum_{i=1}^n |x_i - \mu|$ is the mean deviation of the random sample from its mean. This shall be minimum if μ is the median of the sample. Since

$$\text{Max}(\log L) = \text{Max} \left\{ \sum_{i=1}^n \log \left(\frac{x_i - \mu}{\beta} \right)^2 - \sum_{i=1}^n \left| \frac{x_i - \mu}{\beta} \right| \right\} = M(n \sum_{i=1}^n |x_i - \mu|)$$

it can be shown that $\hat{\mu}$ = Median of the sample if n is odd.
If n is even, then $X_{\frac{n}{2}-1} \leq \hat{\mu} \leq X_{\frac{n}{2}+1}$, where X_j is the j^{th} order statistic.

After obtaining the maximum likelihood estimation of μ as $\hat{\mu}$ and substituting $\hat{\mu}$ in equation (32) the maximum likelihood estimation of β^2 can be obtained by differentiating $\log L$ with respect to β^2 and equating it to zero. After simplification one can obtain

$$\hat{\beta}^2 = \frac{-n + \sum_{i=1}^n (x_i - \hat{\mu})}{2n}$$

$$\text{Also, since } E \left[\frac{-n + \sum_{i=1}^n (x_i - \mu)}{2n} \right]^{-1} = E \left[\frac{n}{2\beta^4} + \frac{1}{2} \frac{\sum_{i=1}^n (x_i - \mu)}{\beta^4} \right]^{-1} = \frac{2\beta^4}{n}$$

The approximate expression to the asymptotic variance of $\hat{\beta}^2$ can be obtained as

$$\text{Asyvar}(\hat{\beta}^2) = E \left[\frac{-\partial^2 \log L}{\partial (\beta^2)^2} \right]^{-1} = \frac{2\beta^4}{n}$$

C. Minimum variance unbiased estimators

In this method the minimum variance unbiased estimators of the location parameter β^2 can be obtained. The general method of estimating μ is to consider the linear combination $l_1 x_1 + l_2 x_2 + \dots + l_n x_n$ of the random

To verify the additive property of the distribution, consider two independent Laplace type bimodal variables X_1 and X_2 with probability density function $f(x_1, \mu_1, \beta_1)$ and $f(x_2, \mu_2, \beta_2)$ respectively and their characteristic functions be $\phi_{X_1}(t)$ and $\phi_{X_2}(t)$. For finding the distribution of $X_1 + X_2$, one can obtain the characteristic function of $X_1 + X_2$. Since X_1 and X_2 are independent,

$$\phi_{X_1+X_2}(t) = \phi_{X_1}(t)\phi_{X_2}(t)$$

Since

$$\phi_{X_j}(t) = \frac{e^{it\mu_j}}{2} \left[\frac{1}{(1+i\beta_j t)^3} + \frac{1}{(1-i\beta_j t)^3} \right], j=1,2$$

then

$$\phi_{X_1+X_2}(t) = \frac{e^{it(\mu_1+\mu_2)}}{2} \left[\frac{1}{(1+i\beta_1 t)^3} + \frac{1}{(1-i\beta_1 t)^3} \right] + \left[\frac{1}{(1+i\beta_2 t)^3} + \frac{1}{(1-i\beta_2 t)^3} \right] \rightarrow 6$$

Since (6) is not in the form of (5) one can say that the distribution of $X_1 + X_2$ is not a Laplace type bimodal distribution. Hence the additive property does not hold good for this distribution. The central moments of this distribution are

$$\mu_{2n} = \frac{1}{4\beta} \int_{-\infty}^{\infty} (x-\mu)^{2n} \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx$$

Making the necessary transformation and by integration, one can get

$$\mu_{2n} = \frac{[(2n+2)!]\beta^{2n}}{2} \text{ and } \mu_{2n+1} = 0 \rightarrow 7$$

Further, the first four central moments of this distribution are

$$\mu_1 = 0, \text{ Variance } \mu_2 = 12\beta^2, \mu_3 = 0, \mu_4 = 360\beta^4$$

Since, the distribution is symmetric, its skewness is zero.

The kurtosis of this distribution is

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{360\beta^4}{12\beta^2} = 2.5 < 3$$

Hence the distribution is platykurtic having the peak less than the Laplace distribution.

The recurrence relation between the central moments is obtained as

$$\mu_{2n} = \frac{[(2n+1)(2n+2)]}{2n(2n-1)} \beta^2 \mu_{2n-2} \rightarrow 8$$

The p^{th} order absolute moments of the distribution are

$$E[|X-\mu|^p] = \frac{1}{4\beta} \int_{-\infty}^{\infty} |x-\mu|^p \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx$$

On simplification, one can have

$$E[|X-\mu|^p] = \left[\frac{(p+2)!}{2} \right] \beta^p \rightarrow 9$$

The Mean deviation about mean μ of the distribution is $MD = 3\beta$

The ratio of the mean deviation to standard deviation is $\frac{3\beta}{\sqrt{12}\beta} = 0.866$

The distribution of the square of the variate $Y = X^2$ can be obtained as

$$f(y) = \frac{1}{4\beta\sqrt{y}} \left(\frac{y}{\beta^2} \right)^2 e^{-\frac{\sqrt{y}}{\beta}}, \quad 0 \leq y < \infty$$

For obtaining the sampling distribution, assume $\mu = 0$ without loss of generality. Let $x_1, x_2, \dots, x_n$ be a sample drawn from the distribution

Case 2: $\mu = 1, \beta = 1$

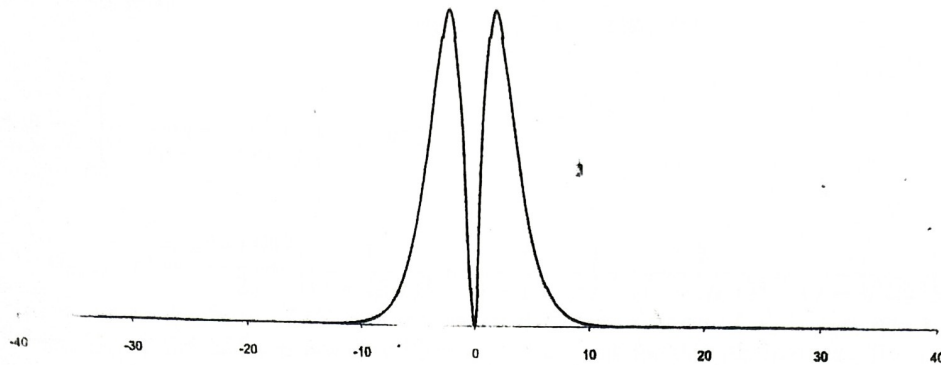


Fig 1
Various shapes of the Laplace type bimodal distribution

III. PROPERTIES OF DISTRIBUTION

The distribution function is

$$F_X(x) = \int_{-\infty}^x \frac{1}{4\beta} \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx = 1 - \left[\frac{1}{2} + \left(\frac{x-\mu}{\beta} \right) + \left(\frac{x-\mu}{\beta} \right)^2 \right] e^{-\left(\frac{x-\mu}{\beta} \right)}, \quad \text{for } x \geq \mu$$

$$= \left[\frac{1}{2} + \left(\frac{x-\mu}{\beta} \right) + \left(\frac{x-\mu}{\beta} \right)^2 \right] e^{-\left(\frac{x-\mu}{\beta} \right)}, \quad \text{for } x < \mu \rightarrow 3$$

Mean of the distribution is

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \frac{1}{4\beta} \int_{-\infty}^{\infty} x \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx$$

On simplification, one can have Mean = μ

The Median ' M ' of this distribution is such that

$$\frac{1}{4\beta} \int_{-\infty}^M x \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx + \frac{1}{4\beta} \int_M^{\infty} x \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx = 1$$

Solving this one can get Median = $M = \mu$. Hence the distribution is symmetric.

Taking the derivative of (1) with respect to x and equating to zero and solving for x , one can get the mode and, in this case, the modes as $\mu - 2\beta$ and $\mu + 2\beta$. These two modes are separated by the distance of 4β . That is the distribution is Bimodal.

The moment generating function of this distribution is

$$M_X(t) = E(e^{tx}) = \frac{1}{4\beta} \int_{-\infty}^{\infty} e^{tx} \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx$$

On simplification, one can get

$$M_X(t) = \frac{e^{t\mu}}{2} \left[\frac{1}{(1+\beta t)^3} + \frac{1}{(1-\beta t)^3} \right] = \frac{e^{t\mu}}{2} \left[\frac{2(1+3\beta^2 t^2)}{(1-\beta^2 t^2)^3} \right] \rightarrow 4$$

The characteristic function of this distribution is

$$\phi_X(t) = \int_{-\infty}^{\infty} e^{itx} f(x) dx = \int_{-\infty}^{\infty} \frac{1}{4\beta} \left(\frac{x-\mu}{\beta} \right)^2 e^{-\left| \frac{x-\mu}{\beta} \right|} dx$$

Using the transformation, $y = x - \mu$, one can get

$$\phi_X(t) = \frac{1}{4\beta} e^{it\mu} \int_{-\infty}^{\infty} e^{ity} \left(\frac{y}{\beta} \right)^2 e^{-\left| \frac{y}{\beta} \right|} dy = \frac{e^{it\mu}}{2} \left[\frac{2-6\beta^2 t^2}{(1+\beta^2 t^2)^3} \right] \rightarrow 5$$

5

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CH. S.D. ST. THERESA'S COLLEGE FOR WOMEN

(Affiliated to Adikavi Nannaya University, Rajahmundry)

Department of Management Studies,

Social Science & Women Studies

West Godavari District, Eluru, Andhra Pradesh, India.



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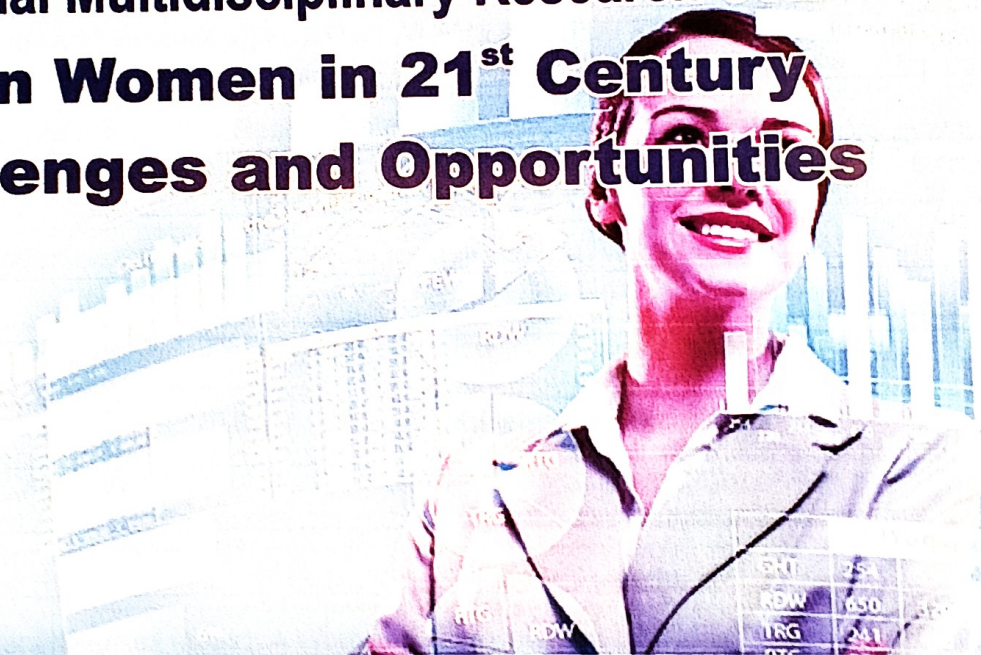


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WOMEN EMPOWERMENT THROUGH THE PATH OF EDUCATION

Dr. D. V. Ramana Murthy¹

D. Sailaja Devi²

D.S. M. V. Vardhan³

Abstract

An African proverb says "If you educate a man, you educate an individual. But if you educate a woman, you educate a nation". Women are the heart of most societies. Regarding of whether they are working or not, mothers are the influential people in child's lives. So empowering them through the "education" forum is crucial and equally important for the better future of next generations. Education has profound effect on girl's and women ability to claim other rights and achieve status in society, such as economic independence and political representation and in other areas of legal reforms, transformation of international economic political relations, action-oriented research and networking. In a country like INDIA where half the population constitutes are women were denied equal opportunities with the hegemonic masculine ideology and domination, as education is the most powerful tool of change for their position in the society. It acknowledges that on the relevance of women education in improving the situation of women in the short term and in emancipating them in long run. Regardless of whether they live in wealthy or poor country, nothing has much impact on a child's future well-being as their mother's level of education. It reminds us of the value of education for lifting nation out of instability and providing a more promising future to their people. This paper throws light on that part of empowerment of girl/women which ensures their accessibility to modern development facilities and extends their participation in social, economic and political process and decision making through the path of enlightenment i.e. "EDUCATION"

Key words – Empowerment, Education, Women, Enlightenment

Introduction:

The significant contribution made by women for the progress of human civilization is no less than that of men, but it has been acknowledged only recently. In India, due to patriarchal society women have always been considered as playing secondary role. Women are at the heart of most societies. Regardless of whether they are working or not, mothers are very influential people in children's lives. Educating girl/women is one of the most important investments that any country can make in its own future.

History has proved that Indian women are capable of becoming rulers and administrators; they are wonderful organizer and are tough fighters. However this has not been highlighted although down the ages women have played a vital role in the family, society, polity, economy and culture. The pages of our history reveal the achievements of a number of great women from different areas of life who have made valuable contributions to the making of INDIA.

Education has a profound effect on girls' and women ability to claim other rights and achieve status in society, such as economic independence and political representation. As they set themselves as examples to demonstrate for their family, having an education can make an enormous difference to a women chances of finding well-paid work, raising a healthy family and preventing the spread of diseases.

Current Education Status Of Women:

Literacy and level of education are basic indicators of the level of development achieved by a society. Spread of literacy is generally associated with important traits of modern civilization such as modernization, urbanization, industrialization, communication and commerce. Literacy forms an important input in overall development of individuals enabling them to comprehend their social, political and cultural environment better and respond to it appropriately.

Higher levels of education and literacy lead to a greater awareness and also contributes in improvement of economic and social conditions. It acts as a catalyst for social upliftment enhancing the returns on investment made in almost every aspect of development effort, be it population control, health, hygiene, environmental degradation control, employment of weaker sections of the society.

In 2011 the percentage of educated persons increased to 74.04% of which male percentage 82.14% and female was 65.46% respectively. Literacy rate among rural women is only 58.8 percentage as per 2011 census. Female literacy at a national average was of 65.46 % whereas the male literacy was 82.14%. Within the Indian states, Kerala has shown the highest literacy rates of 90.02% whereas Bihar averaged lower than 50% literacy, the lowest in India. From the following table given below

¹. Associate Professor & Statistlan & Head of the Department, S.K.V.T. College Rajahmahendravaram

². Research Scholar, Pondicherry University, Pondicherry, India

³. Student, B.Tech, Vishnu University of Technology, Bhimavaram

we can see that India had only 18% literate population at the commencement of the Indian constitution which has become 74.04% in the census of 2011 which means literacy rate in India has been reported as 74.04% with a 14% increase to that in 2001, whereas the hike is maximum for rural women at 26% in the last decade. The female literacy levels according to the Literacy Rate 2011 census are 65.46% whereas the male literacy rate is over 80%.

Table - 1 : Literacy Rate in India

Year	Rural			Urban			Combined		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
1951	4.87	19.02	12.1	22.33	45.6	34.59	8.86	27.15	18.32
1961	10.1	34.3	22.5	40.5	66	54.4	15.35	40.4	28.31
1971	15.5	48.6	27.9	48.8	69.8	60.2	21.97	45.96	34.45
1981	21.7	49.6	36	56.3	76.7	67.2	29.76	56.38	43.57
1991	30.17	56.96	36	64.05	81.09	67.2	39.29	64.13	52.21
2001	46.7	71.4	59.4	73.2	86.7	80.3	53.67	75.26	64.83
2011	58.75	78.57	67.8	79.92	89.67	84.1	65.46	82.14	74.04
% Increase in 2011 over 2001	26%	10%	14%	9%	3%	5%	22%	9%	14%

Source : Census Of India, office of Registrar, India

This is still very pitiable condition as the population of literate women is still hovering around 65% of the total population of the country. Percentage of literate women in the country was just 9% in 1951.

Hurdles for Girls/women Education in Present Scenario:

In many communities & places of our country in both the developed and the developing, we can take it for granted that their Girls/women receiving a quality education appears to be beyond reach.

There are five main challenges we identify that make it difficult for girls/women to access education.

These include:

- The cost of education – ensuring that communities, parents and children can afford schooling;
- Poor school environments – ensuring that girls have access to a safe school environment;
- The weak position of women in society ensuring that society and parents value the education of girls;
- Conflict – ensuring that children who are excluded due to conflict have access to schooling; and
- Social exclusion & social practices (evils) – ensuring girls are not disadvantaged on the basis of caste, ethnicity, religion or disability. Child marriages, dowry system and bonded labour systems also fall in this group. These challenges are not exhaustive, but they are recurrent themes in our country. They constitute additional hurdles girls/women need to overcome to benefit from basic education. As common people; we need to support fellow mates in meeting these challenges. Ours supporting role, may not be a leading role but it can definitely make progress further.

Table - 2 : Percentage distribution of current enrolment status of persons (aged 5-29 years)

Status	Rural		Urban		Rural + Urban	
	Male	Female	Male	Female	Male	Female
Currently not Enrolled	39.4	45.1	41.5	44.2	40	44.8
Currently Enrolled						
Attending	58.7	53	57	54.6	58.2	53.5
Not Attending	1.4	1.2	1.3	1.1	1.4	1.2
All (incl. N.R.)	100	100	100	100	100	100

Source: NSS 71st Round 2014
N.R. stands for not reported

Table - 3 : Per 1000 distribution of never-enrolment (for persons aged 5-29 years) by reasons for non-enrolment:

Major Reasons	Rural		Urban		Rural + Urban	
	Male	Female	Male	Female	Male	Female
Not interested in education	332	270	295	271	325	270
Financial constraints	215	163	328	300	236	185
Engaged in domestic activities	48	234	38	134	46	218
Engaged in economic activities	89	17	69	11	85	16
School is far off	18	29	4	17	16	27
Marriage		6		4		6
No tradition in the community	29	67	22	63	28	67
Other reasons*	268	213	243	200	263	211
All	1000	1000	1000	1000	1000	1000

Source: NSS 71st Round 2014

*include (for all): timings of educational institution not suitable, language/medium of instruction used unfamiliar, inadequate number of teachers, quality of teachers not satisfactory and others; For girl students only, additionally: non-availability of female teacher, non-availability of girls' toilet.

Enhancing Education Empowerment For Girls/Women in following ways:

In particular we need to support them to have in place at least the essential elements of basic education for girls/women which are as follows:

- Schools – is a school within a reasonable distance; does it have proper facilities for girls; is it a safe environment and commute; is it free of violence? If not, parents are unlikely to ever send their daughter to school.
- Teachers – is there a teacher; are they skilled; do they have appropriate teaching materials? Is it a female teacher? Are there policies to recruit teachers from minority communities? If not, girls may not learn as much at school and drop out.
- Students – is she healthy enough; does she feel safe; is she free from the burden of household chores or the need to work to supplement the family income; is there a water source close by? If not, she may never have a chance to go to school.
- Families – does she have healthy parents who can support a family; does her family value education for girls; can her family afford the cost of schooling? If not, economic necessity may keep her at home.
- Societies – will the family's and the girl's standing in the community rise with an education; will new opportunities open up? If not, an education may not be in the family's interest.
- Governments – does the government provide adequate resources to offer sufficient school places; do salaries reach the teachers; do teachers receive quality training; is the government drawing in other agencies to maximise the provision of schooling; is there a clear strategy and budget based on the specific situation faced by girls? If not, the conditions above are unlikely to be fulfilled.

Table - 3 : Percentage distribution of persons (age 5-29 years) who dropped out/discontinued education by age of dropping out / discontinuance:

Age-group of dropping out/ discontinuance	Rural			Urban		
	Male	Female	Person	Male	Female	Person
5-15	58.1	62.7	60.3	45	41.4	43.3
16-24	41.5	36.7	39.2	53.6	57.4	55.4
25-29	0.4	0.6	0.5	1.4	1.2	1.3
All	100	100	100	100	100	100

Source: NSS 71st Round 2014

Persons, who didn't enroll in a particular level, after completing the previous level, are termed as discontinued whereas those who enrolled/attended a specific level but didn't complete that level successfully are called drop-outs. In this document, drop-outs and discontinued are treated together.

Table - 4 : Per 1000 distribution of dropouts/discontinuance (for persons aged 5-29 years) by reasons for dropping out/discontinuance:

Major Reasons	Rural		Urban		Rural + Urban	
	Male	Female	Male	Female	Male	Female
Not interested in education	251	162	208	143	238	156
Financial constraints	236	154	237	149	236	152
Engaged in domestic activities	59	329	24	231	48	297
Engaged in economic activities	299	39	336	69	310	49
School is far off	6	42	2	18	5	34
Unable to cope up with studies	55	51	53	36	54	46
Completed desired level/class	45	43	83	113	57	65
Marriage		124		171		139
Other reasons*	48	57	58	70	51	62
All	1000	1000	1000	1000	1000	1000

Source: NSS 71st Round 2014

*include (for all): timings of educational institution not suitable, language/medium of instruction used unfamiliar, inadequate number of teachers, quality of teachers not satisfactory, unfriendly atmosphere at school, preparation for competitive examination and others. For girl students only, additionally: non-availability of female teacher, non-availability of girls' toilet.

Role of Education in women Empowerment

There is a lot of consensus among the society regarding "empowerment" as it has become one of the most widely used development terms. Women's groups, non-governmental development organisations, activists, politicians, governments and international agencies refer to empowerment as one of their goals. Yet it is one of the least understood in terms of how it is to be measured or observed. It is precisely because this word has now been one of the fashionable concepts to include in policies/programmes/projects that there is a need to clarify and come up with tentative definitions. The nature of empowerment renders it difficult to define. On the one hand, it is often referred to as a goal for many development programmes/projects. On the other hand, it can also be conceived as a process that people undergo, which eventually leads to changes.

The concept of Women empowerment is not a recent one. The first year of New Millennium 2001 was declared as "WOMEN EMPOWERMENT YEAR".

New UNESCO data proves education transforms development. It says:

- If all children enjoyed equal access to education, per capita income would increase by 23%

Over 40 years.

- If all women had a primary education, child marriages and child mortality could fall by a sixth and maternal deaths by two-thirds.

UNESCO's new analysis proves that:-

Education empowers women: Educated girls and young women are more likely to know their rights and to have the confidence to claim them.

Education promotes tolerance: Education helps people to understand democracy, promotes the tolerance and trust that underpin it, and motivates people to participate in the political life of their societies.

Education equality improves job opportunities and increases economic growth: If all children, had equal access to education, productivity gains would boost economic growth. Over 40 years, per capita income would be 23 per cent higher in a country with equality in education.

Education is part of the solution to environmental problems: People with more education are more likely to use energy and water more efficiently and to recycle household waste.

Educating women avoids early marriages: -If education is provided to more number of women it will be helpful in avoiding child marriage.

Education saves mother's lives: In some countries, many women still die because of complications during pregnancy and childbirth. Education can prevent these deaths.

Some child diseases are preventable but not without education: Simple solutions, such as malaria nets and clean water, can prevent some of the worst child diseases, but only if mothers are taught to use them.

Education saves children's lives: Education helps women recognize early signs of illness, seek advice and act on it. If all women in poor countries completed primary education, child mortality would drop by a sixth.

Education fights hunger. The devastating impact of malnutrition on children lives is preventable with the help of education.

Conclusion:

To conclude the present scenario, it may be said that woman in the modern hi- tech society which is moving very fast under the shadow of population explosion, conflicts, chaos and corruption can mould the personality of the adolescent and youth in a proper direction and perspective, provided with the woman themselves in power.

There is a positive relationship between education and woman empowerment. In terms of government policies on women's education, it was observed that while there is no explicit discrimination by gender in most places, neither is there a real commitment to provide sustainable programmes for women. There is such a perceived gap between the rhetoric and policies of decision-makers that many of the women considered the policies as simply paying "lip service" to women's concerns. Even in developed states of our country, the proportion of resources that is being allocated to women's needs is small considering the many diverse needs of the women. The fact that many of the decision-makers are men also constrains their appreciation of these needs.

The actual working and living conditions of women also prevent many of them from meaningful participation in women's education programmes/projects. The increasing impoverishment of women makes it necessary for them to focus on income-generating activities simultaneous with the performance of household chores. This therefore limits their time and energy to get involved in education programmes. Many of the women that have been projected to be beneficiaries of development programmes are illiterate, so the issue of literacy as a women's development concern is likewise a priority.

In spite of the above-mentioned problems, the women agreed that, in order to promote women's empowerment, it is necessary to create an environment that will allow women to participate in educational programmes and share the benefits. It was therefore emphasized that while there is a need to set up specific education programmes for women, there is also a necessity to develop forms of education that will sensitize people towards gender discrimination and will raise their acceptance of women's promotion.

What should never be forgotten is that like boy/men, girl/women need to be part in the process lifelong learning. That is true empowerment from "women for development" the trend has to shift focus on "women in development", that will indeed be a "Quality" change for "Equality". While being attracted by modernisation and globalisation we must be confident to say no to marginalization. We are on the precipice of profound change, as we anchor into new-era of enlightenment there arises a new paradigm success which we are seeking for.

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